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Technical Report 167

DATA REDUCTION TECHNIQUES FOR USE WITH A WIND TUNNEL MAGNETIC SUSPENSION AND BALANCE SYSTEM

by

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FOREWORD

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ABSTRACT

The equations relating the forces and moments exerted on a body by the magnetic fields produced by the MIT-NASA Prototype Magnetic Balance are presented. A computer program which will derive the aerodynamic coefficients for a body using these relations is listed along with a sample output. A preliminary procedure for aligning the axis of the magnetic suspension system with a reference axis is detailed. A procedure for determining dynamic-stability derivatives is outlined.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

SYMBOL	
a,b,c	Principal magnetic body axis
A,B,C,D,E,F,G	Calibration constants
B'	Moment of inertia
b'	Separation of position sensor center and center of gravity
C_i	Constants
$C_{L_{\dot{\alpha}}}$	Damping in lift derivative $\frac{\partial Z}{\partial(\frac{1}{2}\dot{w}t^*\rho u_\infty S)}$
C_{L_q}	Damping in lift derivative $\frac{\partial Z}{\partial(\frac{1}{2}qt^*\rho u_\infty{}^2 S)}$
C_{L_α}	Lift curve slope $\frac{\partial Z}{\partial(\frac{1}{2}w\rho u_\infty S)}$
$C_{M_{\dot{\alpha}}}$	Damping in pitch derivative $\frac{\partial M}{\partial(\dot{w}\rho S\ell^2)}$
C_{M_q}	Damping in pitch derivative $\frac{\partial M}{\partial(q\rho u_\infty S\ell^2)}$
C_{M_α}	Pitch moment curve slope $\frac{\partial M}{\partial(w\rho u_\infty S\ell)}$
D_a, D_b, D_c	Demagnetizing factors
$\vec{d}$	Displacement of center of magnetization from the central point of the magnets
d	Displacement of center of rotation and center of gravity
$\vec{F}$	Magnetic body force vector
F_x, F_y, F_z	Magnetic forces in the x,y,z frame

H_a	Applied magnetic field vector
H_x, H_y, H_z	Applied magnetic field in the x,y,z frame
H_a, H_b, H_c	Applied magnetic field in the a,b,c, frame
h	Separation of center of gravity and magnetic moment center
I_{is}	Inner saddle current
I_{os}	Outer saddle current
I_D	Drag current
I_L	Lift current
I_p	Pitch current
I_s	Slip current
I_y	Yaw current
I_x	Magnetizing current
i_B	Nondimensional moment of inertia about y-axis $\frac{B'}{\rho S \ell^3}$
K_T	Magnetic moment constant
K_i	Constants
K_m	Magnetic stiffness in pitch
$K_{m'}$	Magnetic stiffness in lift
ℓ	Characteristic length
$\vec{\bar{m}}$	Average magnetization vector
$\bar{m}_a, \bar{m}_b, \bar{m}_c$	Magnetization components in the a,b,c, frame
$\bar{m}_x, \bar{m}_y, \bar{m}_z$	Magnetization components in the x,y,z frame
[M]	Matrix to transform x,y,z, axes to a,b,c
$[M]^T$	Matrix to transform a,b,c axes to x,y,z
M	Aerodynamic pitching moment
m	Mass of the model

M_m	Magnetic damping in pitch
$M_{m'}$	Magnetic damping in lift
P_M	Magnetic pitching moment
P_L	Lift position signal
P_p	Pitch position signal
$\bar{P}_L$	Amplitude of lift position signal
$\bar{P}_p$	Amplitude of pitch position signal
q	Derivative $(\frac{\partial \theta}{\partial t})$
S	Reference area
s	Laplace variable
$\vec{T}$	Magnetic body torque vector
T_y, T_z	Magnetic torque about x and y axis, resp.
t*	Reference time (ℓ/u_∞)
t	Time
u	Velocity along x axis
u_∞	Undisturbed stream velocity
V	Volume of ferromagnetic body
V'	Voltage from the amplitude ratio measurement system
w	Velocity along c axis
x,y,z	Reference axis system
Z	Lift force
α	Angle of attack $(= \tan^{-1}\frac{w}{u})$
β, γ	Constant phase angles
δ	Ratio of displacement of center of rotation and center of gravity to characteristic length $(\frac{d}{\ell})$
Δ	Indicates a finite difference

ε	Ratio of separation of center of gravity and magnetic moment center to characteristic length $(\frac{h}{\ell})$
θ	Pitch angle
μ	Nondimensional mass $(\frac{m}{\rho S \ell})$
μ_o	Magnetic permeability
ρ	Density of air
ψ	Yaw angle
ω	Driving frequency
$\dot{w}$, $\dot{\theta}$, $\ddot{\theta}$, etc.	Derivatives with respect to time $(\frac{\partial w}{\partial t}, \frac{\partial \theta}{\partial t}, \frac{\partial^2 \theta}{\partial t^2}$, etc.)
∇	Gradient operator
x	Cross product operator

CHAPTER 1

INTRODUCTION

This report covers work undertaken at the M.I.T. Aerophysics Laboratory to extend the data acquisition capability of the M.I.T.-NASA Prototype Magnetic Balance.[1] Data reduction techniques are presented which derive the forces and moments exerted on a body from the currents producing the magnetic fields which suspend the model.

The primary purpose of the magnetic suspension system is the measurement of aerodynamic forces and moments in the absence of support interference. Since no mechanical contact is made with the model, the aerodynamic forces and moments must be determined from the magnetic fields required to support the model. The supporting magnetic fields are related to the electrical currents passing through the windings of the suspension system. These electrical currents are easily measured with suitable accuracy.

The magnetic forces and moments depend also upon the size and shape of the ferromagnetic part of the suspended model and in addition are functions of the position and orientation of the model relative to the suspension magnet structure. These position variables can be measured either by means of the internal position sensing system which is part of the suspension system control loop, or by external and separate monitoring devices.

The data acquisition and reduction problem can be broken down into several steps, as follows:

1. Establishment of the form of the data-reduction equations (relationship of measured variables to magnet currents, model position, model geometry) to magnetic forces and moments.

2. Calibration - measurement of the parameters defined in the data reduction equations. (1, above)

3. Wind-on data acquisition - measurement and recording of magnet currents, model position, plus wind tunnel parameters required to reduce forces and moments to coefficient form.

4. Data reduction - computation of the aerodynamic forces and moments, force and moment coefficients, Mach number, Reynolds number.

5. Tabulation and/or graphing of aerodynamic coefficients.

It is seen that the general form of the data reduction equations must be assumed in advance. If a general multi-dimensional power series form is assumed which includes all significant nonlinear and interaction terms, then the number of parameters or coefficients to be determined in the calibration process may become impractically large. Each coefficient in general will require at least one calibration data point. If on the other hand, more explicit knowledge is available concerning the form of the equations beforehand, then simplification may be obtained.

One of the design objectives of the MIT-NASA prototype magnetic balance system was simplification of the data reduction process. The simplification was achieved through analysis of the basic relationships between magnetic fields and magnetic forces and moments, and synthesis of a magnet arrangement to implement these relationships as simply as possible. As a result, the general form of the data reduction equations is known, and nonlinear and interaction terms are predicted. The number of calibration coefficients is minimized and there is flexibility in the available methods of measuring these coefficients.

Since the variables related to the forces and moments are continuously available electrical signals, it is possible to provide an essentially continuous record of aerodynamic forces and moments by sampling the variables at a sufficient rate, and computing the corresponding forces and moments. Simple or complex model motions are continuously controllable over a limited bandwidth, and by correlation of the computed forces and moments with the model kinematics, unsteady aerodynamic effects (such as damping coefficients) may be evaulated.

This report contains a detailed description of data reduction equations, calibration methods, and a computer program for data reduction. In addition, details concerning a method of determining the aerodynamic pitch damping coefficient, and details of the preliminary alignment procedures used with the M.I.T.-NASA Magnetic Balance System are discussed.

CHAPTER 2

STATIC FORCE AND MOMENT REDUCTION EQUATIONS

The equations relating the magnetic forces and moments on a ferromagnetic body to the applied fields are the following.[2]

The magnetic force vector, $\vec{F}$, is approximately*

$$\vec{F} \cong K_T V(\vec{\bar{m}} \cdot \nabla)\ \vec{H}_A \tag{2.1}$$

and the magnetic torque, $\vec{T}$, is approximately*

$$\vec{T} \cong K_T V(\vec{\bar{m}} \times \vec{H}_A) \tag{2.2}$$

where

V = ferromagnetic model volume

$\vec{\bar{m}}$ = average model magnetization

$\vec{H}_A$ = applied magnetic field flux density

∇ = gradient operator

x = cross product operator

K_T = magnetic moment constant

Considering first the equation for the magnet forces, this equation can be written in matrix form for the frame of the magnetic balance as,

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*Exact equations for $\vec{F}$ and $\vec{T}$ require integration over the volume of the ferromagnetic body.

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = K_T V \begin{bmatrix} \frac{\partial H_x}{\partial x} & \frac{\partial H_x}{\partial y} & \frac{\partial H_x}{\partial z} \\ \frac{\partial H_y}{\partial x} & \frac{\partial H_y}{\partial y} & \frac{\partial H_y}{\partial z} \\ \frac{\partial H_z}{\partial x} & \frac{\partial H_z}{\partial y} & \frac{\partial H_z}{\partial z} \end{bmatrix} \begin{Bmatrix} \bar{m}_x \\ \bar{m}_y \\ \bar{m}_z \end{Bmatrix} \tag{2.3}$$

The average model magnetization $\bar{\vec{m}}$ can be expressed in the body frame (see Fig. 1) as*

$$\bar{m}_a = \frac{H_a}{D_a} \tag{2.4a}$$

$$\bar{m}_b = \frac{H_b}{D_b} \tag{2.4b}$$

$$\bar{m}_c = \frac{H_c}{D_c} \tag{2.4c}$$

where

H_a, H_b, H_c = applied magnetic field strength in the body frame

D_a, D_b, D_c = average demagnetizing factors (related to the shape of the ferromagnetic body (see Ref. 2))

$\bar{m}_a$, $\bar{m}_b$, $\bar{m}_c$ = average model magnetization in the body frame

Eqn. (2.4) may also be written in matrix form as,

$$\begin{Bmatrix} \bar{m}_a \\ \bar{m}_b \\ \bar{m}_c \end{Bmatrix} = \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_c} \end{bmatrix} \begin{Bmatrix} H_a \\ H_b \\ H_c \end{Bmatrix} \tag{2.5}$$

To change from the body frame of reference to the balance frame (see Fig. 1), the body frame will be considered as being rotated an angle ψ in the x-y plane, inclined an angle θ above the x-y plane, and rotated an angle ϕ about the resulting longitudinal axis. These three rotations can be expressed as

*The approximation used here requires the product of the demagnetization factor and the magnetic permeability, μ_o, to be large; that is, the error in Eq. 2.4 is of the order of $\left(\frac{1}{\mu_o D_{a,b,c}}\right)$.

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{2.6}$$

where x, y, z = quantities in the balance frame
a, b, c = quantities in the body frame
or after multiplying the matrices

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi\cos\theta & -\sin\theta \\ -\sin\psi\cos\phi+\sin\phi\cos\psi\sin\theta & \cos\psi\cos\phi+\sin\phi\sin\psi\sin\theta & \sin\phi\cos\theta \\ \sin\psi\sin\phi+\cos\phi\cos\psi\sin\theta & -\sin\phi\cos\psi+\cos\phi\sin\psi\sin\theta & \cos\phi\cos\theta \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{2.7}$$

using this result and noting that the inverse matrix for an orthogonal transformation equals the transposed matrix yields

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & -\sin\psi\cos\phi+\sin\phi\cos\psi\sin\theta & \sin\psi\sin\phi+\cos\phi\cos\psi\sin\theta \\ \sin\psi\cos\theta & \cos\psi\cos\phi+\sin\phi\sin\psi\sin\theta & -\sin\phi\cos\psi+\cos\phi\sin\psi\sin\theta \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \tag{2.8}$$

denoting the transformation matrix as [M], eqns. (2.7) and (2.8) become

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = [M] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \quad (2.9); \qquad \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [M]^T \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \quad (2.10)$$

using this result the average model magnetization and the applied magnetic fields may be written as,

$$\begin{Bmatrix} \bar{m}_x \\ \bar{m}_y \\ \bar{m}_z \end{Bmatrix} = [M]^T \begin{Bmatrix} \bar{m}_a \\ \bar{m}_b \\ \bar{m}_c \end{Bmatrix} \quad (2.11); \qquad \begin{Bmatrix} H_a \\ H_b \\ H_c \end{Bmatrix} = [M] \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \quad (2.12)$$

combining eqns. (2.5), (2.11) and (2.12) yields

$$\begin{Bmatrix} \bar{m}_x \\ \bar{m}_y \\ \bar{m}_z \end{Bmatrix} = [M]^T \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_c} \end{bmatrix} [M] \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \qquad (2.13)$$

therefore, from eqns. (2.3) and (2.13), the expression for the magnetic forces becomes,

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = K_T V \begin{bmatrix} \frac{\partial H_x}{\partial x} & \frac{\partial H_x}{\partial y} & \frac{\partial H_x}{\partial z} \\ \frac{\partial H_y}{\partial x} & \frac{\partial H_y}{\partial y} & \frac{\partial H_y}{\partial z} \\ \frac{\partial H_z}{\partial x} & \frac{\partial H_z}{\partial y} & \frac{\partial H_z}{\partial z} \end{bmatrix} [M]^T \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_c} \end{bmatrix} [M] \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \qquad (2.14)$$

The torque can be expressed in terms of the magnetic fields using the expression for model magnetization. First, expand eqn. (2.2) to yield (rolling torque excluded)

$$\begin{aligned} T_y &= K_T V \ (\bar{m}_z H_x - \bar{m}_x H_z) \\ T_z &= K_T V \ (\bar{m}_x H_y - \bar{m}_y H_x) \end{aligned} \qquad (2.15)$$

or, in matrix form using eqns. (2.13) and (2.2)

$$\begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix} = K_T V \quad [M]^T \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_c} \end{bmatrix} [M] \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \times \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \qquad (2.16)$$

<u>Relations among Gradients*</u>

The magnetic field gradients are related through Maxwell's equations. In the region of interest there are no electric

* Some of the material here is taken from Ref. 2. and is included for clarification.

currents (and no distributed poles) which results in the following:

$$\vec{\nabla} \times \vec{H} = 0 \tag{2.17}$$

i.e.,

$$\frac{\partial H_x}{\partial y} = \frac{\partial H_y}{\partial x} \tag{2.18a}$$

$$\frac{\partial H_x}{\partial z} = \frac{\partial H_z}{\partial x} \tag{2.18b}$$

$$\frac{\partial H_y}{\partial z} = \frac{\partial H_z}{\partial y} \tag{2.18c}$$

Also

$$\vec{\nabla} \cdot \vec{H} = 0 \tag{2.19}$$

i.e.,

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \tag{2.20}$$

Several assumptions become necessary at this point in order to continue the development. These are,

1. In the wind tunnel frame of reference, at the center of symmetry of the balance system, by virtue of the magnet geometry, the magnetic field and field gradient components are related to the applied magnet currents as follows:

$$H_x = K_1 I_x \tag{2.21a}$$

$$H_y = K_5 I_y \tag{2.21b}$$

$$H_z = K_3 I_p \tag{2.21c}$$

$$\frac{\partial H_x}{\partial x} = K_2 I_D \tag{2.21d}$$

$$\frac{\partial H_z}{\partial x} = \frac{\partial H_x}{\partial z} = K_4 I_L \tag{2.21e}$$

$$\frac{\partial H_y}{\partial x} = \frac{\partial H_x}{\partial y} = K_6 I_s \tag{2.21f}$$

where

I_x = magnetizing current
I_p = pitch current
I_y = yaw current
I_d = drag current
I_L = lift current
I_s = slip current
$K_1, K_3, K_5, K_2, K_4, K_6$ = constants.

2. The gradient terms $\partial H_z/\partial_y = \partial H_y/\partial_z = 0$

3. The ferromagnetic body is axisymmetric.

i.e., $D_b = D_c$

4. The magnet current controlling the gradient term $\partial H_x/\partial_x$ also controls the coupled terms $\partial H_y/\partial_y$ and $\partial H_z/\partial_z$. Since the particular magnet system controlling $\partial H_x/\partial_x$ is axisymmetric (about x-axis) then these gradient terms are reduced as follows:

$$\frac{\partial H_y}{\partial y} = \frac{\partial H_z}{\partial z} = -\frac{1}{2}\frac{\partial H_x}{\partial x}$$

Combining the preceding results with equations (2.14) and (2.16) yields

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = K_T V \begin{bmatrix} K_2 I_D & K_6 I_s & K_4 I_L \\ K_6 I_s & -\frac{1}{2}K_2 I_D & 0 \\ K_4 I_L & 0 & -\frac{1}{2}K_2 I_D \end{bmatrix} [M]^T \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_b} \end{bmatrix} [M] \begin{Bmatrix} K_1 I_x \\ K_5 I_y \\ K_3 I_p \end{Bmatrix} \tag{2.22}$$

and

$$\begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix} = K_T V \; [M]^T \begin{bmatrix} \frac{1}{D_a} & 0 & 0 \\ 0 & \frac{1}{D_b} & 0 \\ 0 & 0 & \frac{1}{D_c} \end{bmatrix} [M] \begin{Bmatrix} K_1 I_x \\ K_5 I_y \\ K_3 I_p \end{Bmatrix} \times \begin{Bmatrix} K_1 I_x \\ K_5 I_y \\ K_3 I_p \end{Bmatrix} \tag{2.23}$$

If now equations (2.22) and (2.23) are expanded, the resulting force equations are (assuming roll angle equals zero, i.e., $\phi = 0$)

$$
\begin{aligned}
F_x = \frac{K_T V}{D_b} \{ & K_2 I_D [K_1 I_x (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \cos^2\theta \cos^2\psi) \\
& -K_5 I_y (1 - \frac{D_b}{D_a}) \cos^2\theta \sin\psi \cos\psi \\
& +K_3 I_p (1 - \frac{D_b}{D_a}) \sin\theta \cos\theta \cos\psi] \\
& + K_6 I_s [-K_1 I_x (1 - \frac{D_b}{D_a}) \cos^2\theta \sin\psi \cos\psi \\
& +K_5 I_y (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \sin^2\psi \cos^2\theta) \\
& +K_3 I_p (1 - \frac{D_b}{D_a}) \sin\theta \sin\psi \cos\theta] \\
& + K_4 I_L [K_1 I_x (1 - \frac{D_b}{D_a}) \sin\theta \cos\theta \cos\psi \\
& +K_5 I_y (1 - \frac{D_b}{D_a}) \sin\theta \sin\psi \cos\theta \\
& +K_3 I_p (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \sin^2\theta)]\}
\end{aligned} \tag{2.24a}
$$

$$
\begin{aligned}
F_y = \frac{K_T V}{D_b} \{ & K_6 I_s [K_1 I_x (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \cos^2\theta \cos^2\psi) \\
& -K_5 I_y (1 - \frac{D_b}{D_a}) \cos^2\theta \sin\psi \cos\psi \\
& +K_3 I_p (1 - \frac{D_b}{D_a}) \sin\theta \cos\theta \cos\psi] \\
& + \frac{1}{2} K_2 I_D [K_1 I_x (1 - \frac{D_b}{D_a}) \cos^2\theta \sin\psi \cos\psi \\
& -K_5 I_y (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \sin^2\psi \cos^2\theta) \\
& -K_3 I_p (1 - \frac{D_b}{D_a}) \sin\theta \sin\psi \cos\theta]\}
\end{aligned} \tag{2.24b}
$$

$$F_z = \frac{K_T V}{D_b} \{K_4 I_L [K_1 I_x (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \cos^2\theta\cos^2\psi)$$
$$-K_5 I_y (1 - \frac{D_b}{D_a}) \cos^2\theta \sin\psi \cos\psi$$
$$+K_3 I_p (1 - \frac{D_b}{D_a}) \sin\theta\cos\theta\cos\psi]$$
$$- \frac{1}{2} K_2 I_D [K_1 I_x (1 - \frac{D_b}{D_a}) \sin\theta\cos\theta\cos\psi \qquad (2.24c)$$
$$+ K_5 I_y (1 - \frac{D_b}{D_a}) \sin\theta\sin\psi\cos\theta$$
$$+ K_3 I_p (1 - \left\langle 1 - \frac{D_b}{D_a} \right\rangle \sin^2\theta)]\}$$

and the result of expanding the torque equation is

$$T_y = \frac{K_T V}{D_b} \{(K_1^2 I_x^2 - K_3^2 I_p^2)(1 - \frac{D_b}{D_a}) \sin\theta\cos\theta\cos\psi$$
$$+ K_1 K_5 I_x I_y (1 - \frac{D_b}{D_a}) \sin\theta\sin\psi\cos\theta \qquad (2.25a)$$
$$+ K_1 K_3 I_x I_p (1 - \frac{D_b}{D_a})(\cos^2\theta\cos^2\psi - \sin^2\theta)$$
$$+ K_3 K_5 I_p I_y (1 - \frac{D_b}{D_a}) \cos^2\theta\sin\psi \cos\psi\}$$

$$T_z = \frac{K_T V}{D_b} \{K_1 K_5 I_x I_y (1 - \frac{D_b}{D_a})(\sin^2\psi\cos^2\theta - \cos^2\psi\cos^2\theta)$$
$$+ (K_1^2 I_x^2 - K_5^2 I_y^2)(1 - \frac{D_b}{D_a}) \cos^2\theta\sin\psi \cos\psi \qquad (2.25b)$$
$$+K_3 K_5 I_p I_y (1 - \frac{D_b}{D_a}) \sin\theta\cos\theta\cos\psi$$
$$-K_1 K_3 I_x I_p (1 - \frac{D_b}{D_a}) \sin\theta\sin\psi\cos\theta\}$$

If now the following relations are defined

$$A = \frac{K_T V\ K_2 K_1}{D_a} \qquad (2.26a) \qquad D = \frac{K_T V}{D_b} K_1 K_5 \ (1 - \frac{D_b}{D_a}) \qquad (2.26d)$$

$$B = \frac{K_T V\ K_6 K_1}{D_a} \qquad (2.26b) \qquad E = \frac{K_5}{K_1} \qquad (2.26e)$$

$$C = \frac{K_T V\ K_4 K_1}{D_a} \qquad (2.26c) \qquad F = \frac{K_T V}{D_b} K_1 K_3 (1 - \frac{D_b}{D_a}) \qquad (2.26f)$$

$$G = \frac{K_3}{K_1} \qquad (2.26g)$$

these may be substituted into eqns. (2.24) and (2.25) to result in a final form:

$$\begin{aligned}
F_x = {} & A\ I_D I_x \ \{\frac{D_a}{D_b} + (1 - \frac{D_a}{D_b})\ \cos^2\theta\cos^2\psi\} \\
& + A\ E\ I_D I_y \ (1 - \frac{D_a}{D_b})\ \cos^2\theta\sin\psi\ \cos\psi \\
& - A\ G\ I_D I_p \ (1 - \frac{D_a}{D_b})\ \sin\theta\cos\theta\cos\psi \\
& + B\ I_s I_x \ (1 - \frac{D_a}{D_b})\ \cos^2\theta\sin\psi\ \cos\psi \\
& + B\ E\ I_s I_y \ \{\frac{D_a}{D_b} + (1 - \frac{D_a}{D_b})\ \sin^2\psi\cos^2\theta\} \qquad (2.27a) \\
& - B\ G\ I_s I_p \ (1 - \frac{D_a}{D_b})\ \sin\theta\sin\psi\cos\theta \\
& - C\ I_L I_x \ (1 - \frac{D_a}{D_b})\ \sin\theta\cos\theta\cos\psi \\
& - C\ E\ I_L I_y \ (1 - \frac{D_a}{D_b})\ \sin\theta\sin\psi\cos\theta \\
& + C\ G\ I_L I_p \ \{\frac{D_a}{D_b} + (1 - \frac{D_a}{D_b})\ \sin^2\theta\}
\end{aligned}$$

$$
\begin{aligned}
F_y = {} & B\, I_s I_x \left\{\frac{D_a}{D_b} + \left(1 - \frac{D_a}{D_b}\right) \cos^2\theta\cos^2\psi\right\} \\
& + B\, E\, I_s I_y \left(1 - \frac{D_a}{D_b}\right) \cos^2\theta\sin\psi\cos\psi \\
& - B\, G\, I_s I_p \left(1 - \frac{D_a}{D_b}\right) \sin\theta\cos\theta\cos\psi \\
& - \frac{1}{2} A\, I_D I_x \left(1 - \frac{D_a}{D_b}\right) \cos^2\theta\sin\psi\cos\psi \\
& - \frac{1}{2} A\, E\, I_D I_y \left\{\frac{D_a}{D_b} + \left(1 - \frac{D_a}{D_b}\right) \sin^2\psi\cos^2\theta\right\} \\
& + \frac{1}{2} A\, G\, I_D I_p \left(1 - \frac{D_a}{D_b}\right) \sin\theta\sin\psi\cos\theta
\end{aligned}
\tag{2.27b}
$$

$$
\begin{aligned}
F_z = {} & C\, I_L I_x \left\{\frac{D_a}{D_b} + \left(1 - \frac{D_a}{D_b}\right) \cos^2\theta\cos^2\psi\right\} \\
& + C\, E\, I_L I_y \left(1 - \frac{D_a}{D_b}\right) \cos^2\theta\sin\psi\cos\psi \\
& - C\, G\, I_L I_p \left(1 - \frac{D_a}{D_b}\right) \sin\theta\cos\theta\cos\psi \\
& + \frac{1}{2} A\, I_D I_x \left(1 - \frac{D_a}{D_b}\right) \sin\theta\cos\theta\cos\psi \\
& + \frac{1}{2} A\, E\, I_D I_y \left(1 - \frac{D_a}{D_b}\right) \sin\theta\sin\psi\cos\theta \\
& - \frac{1}{2} A\, G\, I_D I_p \left\{\frac{D_a}{D_b} + \left(1 - \frac{D_a}{D_b}\right) \sin^2\theta\right\}
\end{aligned}
\tag{2.27c}
$$

$$
\begin{aligned}
T_y = {} & F\, I_x I_p \left(\cos^2\theta\cos^2\psi - \sin^2\theta\right) \\
& + \left(\frac{F}{G} I_x^{\,2} - F\, G\, I_p^{\,2}\right) \sin\theta\cos\theta\cos\psi \\
& + F\, E\, I_p I_y \cos^2\theta\sin\psi\cos\psi \\
& + D\, I_x I_y \sin\theta\sin\psi\cos\theta
\end{aligned}
\tag{2.28a}
$$

$$T_z = D\, I_x I_y \, (\sin^2\psi\cos^2\theta - \cos^2\theta\cos^2\psi)$$

$$+ \left(\frac{D}{E}\, I_x{}^2 - D\, E\, I_y{}^2\right) \cos^2\theta\sin\psi\cos\psi$$

$$+ D\, G\, I_p I_y \sin\theta\cos\theta\cos\psi \qquad (2.28b)$$

$$- F\, I_x I_p \sin\theta\sin\psi\cos\theta$$

The preceding equations are the relations used in practice to determine the forces and moments on the ferromagnetic body if there are no displacements from the central point of the magnetic balance.

Determination of Constants

For zero θ and ψ (pitch and yaw angle) and zero displacements from the balance central point, eqns. (2.27) and (2.28) become

$$F_x = A\, I_x I_D + \frac{D_a}{D_b}\,(B\, E\, I_y I_s + C\, G\, I_p I_L) \qquad (2.29a)$$

$$F_y = B\, I_x I_s - \frac{1}{2}\,\frac{D_a}{D_b}\, A\, E\, I_y I_D \qquad (2.29b)$$

$$F_z = C\, I_x I_L - \frac{1}{2}\,\frac{D_a}{D_b}\, A\, G\, I_p I_D \qquad (2.29c)$$

$$T_z = -\, D\, I_x I_y \qquad (2.29d)$$

$$T_y = F\, I_x I_p \qquad (2.29e)$$

If, furthermore, forces are applied sequentially in the x, y and z directions, the second term in the force equations may be considered to be equal to zero. The constants may now be written in terms of applied forces as

$$A = \left.\frac{\Delta F_x}{\Delta I_x I_D}\right|_{\substack{\theta=\psi=0 \\ I_y I_s,\, I_p I_L = \text{constant}}} \qquad (3.30a)$$

$$B = \left. \frac{\Delta F_y}{\Delta I_x I_s} \right|_{\substack{\theta=\psi=0 \\ I_y I_D=\text{constant}}} \tag{2.30b}$$

$$C = \left. \frac{\Delta F_z}{\Delta I_x I_L} \right|_{\substack{\theta=\psi=0 \\ I_p I_D=\text{constant}}} \tag{2.30c}$$

and in terms of applied torques as

$$D = - \left. \frac{\Delta T_z}{\Delta I_x I_y} \right|_{\theta=\psi=0} \tag{2.31a}$$

$$F = \left. \frac{\Delta T_y}{\Delta I_x I_p} \right|_{\theta=\psi=0} \tag{2.31b}$$

The constants, E and G, which are a property of the magnet configuration are determined by changing the model's orientation, and measuring the corresponding changes in yaw and pitch current. Following a procedure developed in Ref. 3 and noting that the ΔT_z and ΔT_y are equal to zero, the constants, E and G are determined as follows:

$$E \Big|_{\substack{\theta=0 \\ \Delta T_y=\Delta T_z=0}} = - \frac{c}{b} - \frac{ac^2}{b^3} - 2 \frac{a^2c^3}{b^5} - \ldots \tag{2.32a}$$

and

$$G \Big|_{\substack{\psi=0 \\ \Delta T_y=\Delta T_z=0}} = - \frac{c'}{b'} - \frac{a'c'^2}{b'^3} - 2 \frac{a'^2c'^3}{b'^5} - \ldots \tag{2.32b}$$

where

$$a = I_{y_1}^2 \frac{\sin 2\psi_1}{2} \quad (2.33a) \qquad a' = I_{p_1}^2 \frac{\sin 2\theta_1}{2} \quad (2.34a)$$

$$b = I_{yo} I_x - I_x I_{y_1} \cos 2\psi_1 \quad (2.33b) \qquad b' = I_{p_o} I_x - I_{p_1} I_x \cos 2\theta_1 \quad (2.34b)$$

$$c = - I_x^2 \frac{\sin 2\psi_1}{2} \quad (2.33c) \qquad c' = - I_x^2 \frac{\sin 2\theta_1}{2} \quad (2.34c)$$

and

$$I_y \Big|_{\psi=\psi_1} = I_{y_1} \quad (2.35a) \qquad I_p \Big|_{\theta=\theta_1} = I_{p_1} \quad (2.35c)$$

$$I_y \Big|_{\psi=0} = I_{y_o} \quad (2.35b) \qquad I_p \Big|_{\theta=0} = I_{p_o} \quad (2.35d)$$

The ratio of the demagnetizing factors (D_a/D_b) can be determined according to a procedure outlined in Ref. 2.

Corrections due to an Offset of the Center of Magnetization and the Magnetic Center of the Balance

If the center of magnetization of the model is displaced from the magnetic center of the balance, the gradient fields will contribute to the uniform fields. To incorporate this correction expand the total applied field in a Taylor series of first order to give

$$\vec{H}_A = \vec{H}_A(0) + (\vec{d} \cdot \vec{\nabla}) \vec{H}_A \qquad (2.36)$$

where

$\vec{H}_A$ = total applied field

$\vec{d}$ = displacement of center of magnetization from the central point of the magnets

$\vec{\nabla}$ = field gradient operator

Eqn. (2.36) can be expanded to yield

$$H_x = H_x(0) + \frac{\partial H_x}{\partial x}\,\bar{x} + \frac{\partial H_x}{\partial y}\,\bar{y} + \frac{\partial H_x}{\partial z}\,\bar{z} \tag{2.37a}$$

$$H_y = H_y(0) + \frac{\partial H_y}{\partial x}\,\bar{x} + \frac{\partial H_y}{\partial y}\,\bar{y} + \frac{\partial H_y}{\partial z}\,\bar{z} \tag{2.37b}$$

$$H_z = H_z(0) + \frac{\partial H_z}{\partial x}\,\bar{x} + \frac{\partial H_z}{\partial y}\,\bar{y} + \frac{\partial H_z}{\partial z}\,\bar{z} \tag{2.37c}$$

Now,

$$H_x(0) = K_1 I_{x_o} \tag{2.38a}$$

$$H_y(0) = K_5 I_{y_o} \tag{2.38b}$$

$$H_z(0) = K_3 I_{p_o} \tag{2.38c}$$

Using the relations between the currents and fields, these equations become,

$$I_x = I_{x_o} + \frac{K_2}{K_1}\,I_D\,\bar{x} + \frac{K_6}{K_1}\,I_s\bar{y} + \frac{K_4}{K_1}\,I_L\bar{z} \tag{2.39a}$$

$$I_p = I_{p_o} + \frac{K_4}{K_3}\,I_L\,\bar{x} - \frac{1}{2}\,\frac{K_2}{K_3}\,I_D\bar{z} \tag{2.39b}$$

$$I_y = I_{y_o} + \frac{K_6}{K_5}\,I_s\,\bar{x} - \frac{1}{2}\,\frac{K_2}{K_5}\,I_D\bar{y} \tag{2.39c}$$

where
$$\frac{K_2}{K_1} = \frac{AG}{F}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40a}$$

$$\frac{K_4}{K_3} = \frac{C}{F}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40b}$$

$$\frac{K_6}{K_1} = \frac{BG}{F}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40c}$$

$$\frac{K_4}{K_1} = \frac{CG}{F}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40d}$$

$$\frac{K_2}{K_3} = \frac{A}{F}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40e}$$

$$\frac{K_6}{K_5} = \frac{B}{D}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40f}$$

$$\frac{K_2}{K_5} = \frac{A}{D}\left(\frac{D_a}{D_b} - 1\right) \tag{2.40g}$$

I_x, I_p, I_y = effective field currents

$I_{x_o}, I_{p_o}, I_{y_o}$ = measured field currents

To determine the actual displacements it is desirable to curve fit the calibration data to the expected form of the relationship between the currents and the forces. For example, for zero pitch and yaw angle the lift equation becomes,

$$F_z = CI_L I_{x_o} \tag{2.41}$$

If the correction to magnetizing current is applied the equation expands to

$$F_z = CI_L(I_{x_o} + \frac{K_2}{K_1} I_D \bar{x} + \frac{K_6}{K_1} I_s \bar{y} + \frac{K_4}{K_1} I_L \bar{z}) \tag{2.42}$$

For the case in which the model is loaded purely in lift,

$$\Delta\frac{K_2}{K_1} I_D \bar{x} = \Delta\frac{K_6}{K_1} I_s \bar{y} = 0 \tag{2.43}$$

Therefore, eqn. (2.42) becomes (assuming a tare lift load)

$$F_z - F_{z_o} = CI_L I_{x_o} + C \frac{K_4}{K_1} \bar{z} I_L{}^2 \tag{2.44}$$

Now, taking three data points, eqn. (2.44) may be solved in matrix notation as follows,

$$\begin{Bmatrix} F_{z_1} \\ F_{z_2} \\ F_{z_3} \end{Bmatrix} = \begin{bmatrix} I_{L_1} I_{x_o} & I_{L_1}^2 & 1 \\ I_{L_2} I_{x_o} & I_{L_2}^2 & 1 \\ I_{L_3} I_{x_o} & I_{L_3}^2 & 1 \end{bmatrix} \begin{Bmatrix} C \\ C \frac{K_4}{K_1} \bar{z} \\ F_{z_o} \end{Bmatrix} \tag{2.45}$$

or

$$\bar{F}_z = [N] \; \bar{c} \tag{2.46}$$

the solution is now obtained by inverting the matrix [N] so that

$$\begin{Bmatrix} C \\ C \frac{K_4}{K_1} \bar{z} \\ F_{z_o} \end{Bmatrix} = [N]^{-1} \begin{Bmatrix} F_{z_1} \\ F_{z_2} \\ F_{z_3} \end{Bmatrix} \tag{2.47}$$

The plot of $C \frac{K_4}{K_1} \bar{z}$ determined by eqn. (2.47) versus the lift position (z) for the delta wing models, shown in Fig. 2, demonstrates the validity of this correction to the calibration equations. The drag and side force calibration equations may be modified to account for $\bar{x}$ and $\bar{y}$ displacements in a similar manner.

These corrections for $\bar{x}$, $\bar{y}$ and $\bar{z}$ are small and may be neglected if the linear approximation to the drag, side force, and lift calibration data respectively, is accurate over the range tested, as this is an indication that the ratio of the uniform fields to the gradient fields is large, or that displacements are small. The balance should be capable of accuracies of better than 0.5%.

CHAPTER 3

APPLICATION OF FORCE AND MOMENT REDUCTION EQUATIONS

Due to the rather lengthy calculations involved in deriving the forces and moments from the magnet currents using the relations presented in Chapter 2, a computer has been used to transform the magnet currents into the force and moment information they represent. The computer program used presently is listed in Appendix A. In addition to computing the forces and moments, this program computes the aerodynamic coefficients corresponding to the particular forces and moments calculated for a given set of conditions for the wind tunnel used. At present, a supersonic and a subsonic tunnel have been adapted for use with the prototype magnetic suspension system and the computer program will accept data on the tunnel conditions of either and deduce the information necessary to compute the aerodynamic coefficients. The two wind tunnels are the 4" x 4", Mach 4.24 open jet tunnel and the dynamic pressure simulator (wind speed variable from 0 to 500 ft/sec) at the M.I.T. Aerophysics Laboratory. Typical output of the computer program is shown in Appendix B.

Experimental Results

Two procedures have been applied to determine the validity of the expressions for the forces and moments presented in Chapter 2. The first consisted of hanging weights on a suspended model and measuring the additional magnet currents required to balance the applied forces. For this procedure

Eqns. (2.30a-c and 2.31a,b) predict that if a force or moment is applied to a body with the body axis of symmetry parallel to the axis of symmetry of the magnetic balance and the center of magnetization of the body coincident with the magnetic center of the balance, then the force or moment is linearly related to the corresponding magnet current (at constant magnetizing current). A plot of the applied axial force versus the product of the drag current and the magnetizing current is shown in Fig. 3 as a typical result. The points deviate a maximum of 0.5% from the assumed linear relation (the actual body tested was a blunted 6° half angle cone). In general, forces may be measured to ± 0.01 oz. In some instances, it may be necessary to apply the displacement of center of magnetization corrections discussed in Chapter 2 to attain this accuracy.

The accuracy of moment measurement is affected more strongly by angle of attack variations than are the force measurements. In explanation, the fields producing the moments are also used to control the orientation of the model. The effect is similar to that of a torsional spring. To determine the torque exerted by the spring, the difference in the angles at each end of the spring is determined. In an analogous manner, the method of determining the torque exerted by the magnetic fields may be described as determining the angular difference of the applied magnetic field vector and the axis of symmetry of the model. Therefore, the portion of the field causing the orientation of the model must be deducted from the total field. The equations presented in Chapter 2 for moment measurement take this effect into account. However, if an uncertainty in model orientation exists, then an uncertainty in the measured moment will exist. The magnitude of the uncertainty is proportional to the magnitude of the magnetizing field squared since the angle of attack sensitivity is proportional to the ratio of the moment producing field to the magnetizing field. In explanation, eqn. (2.28a) becomes for zero moments and zero yaw angle

$$\Delta T_y = 0 = FI_xI_p(\cos^2\theta - \sin^2\theta) + (\frac{F}{G}I_x{}^2 - FG\ I_p{}^2)\sin\theta\cos\theta \qquad (3.1)$$

or after some manipulation,

$$-1 = \frac{1}{2}(\frac{1}{G}\ \frac{I_x}{I_p} - G\frac{I_p}{I_x})\ \frac{\sin 2\theta}{\cos 2\theta} \qquad (3.2)$$

for small pitch angles (which usually implies small pitch currents), eqn. (3.2) becomes

$$-1 \cong \theta\cdot\frac{1}{G}\ \frac{I_x}{I_p} \qquad (3.3)$$

$$\text{or } \theta \cong\ -G\ \frac{I_p}{I_x} \qquad (3.4)$$

Since the moment sensitivity from the calibration equation (eqn. 2.32b) is

$$T_y = F\ I_xI_p \qquad (3.5)$$

the sensitivity of the moment relation to angle of attack error can be expressed as

$$\frac{\Delta T_y}{\Delta\theta} \cong -\ \frac{F}{G}\ I_x{}^2 \qquad (3.6)$$

Plots of the applied moment versus the product of the pitch current and the magnetizing current are shown in figs. 4 and 5 for two configurations (a delta wing and a blunted 6° half angle cone) which required a different magnetizing current. In addition, the error band resulting from an angle of attack uncertainty of $\pm$ 0.1° is indicated. However, reducing the magnetizing current to improve moment resolution will not be feasible in all circumstances, since this would reduce the overall force and moment range. This is because more current would be required for the other magnets to produce the same forces and moments at the lower magnetizing current.

The second approach used to determine the validity of force, moment and current relations to the magnet currents involved the use of a pneumatic calibration rig (see Ref. 4). Briefly, the calibration rig is an array of thrust and journal air bearings capable of supporting a range of loads. Loading the pneumatic calibration rig causes changes in the air gap between the thrust and journal bearings, resulting in a changed pressure distribution in the air gap. Therefore, the pressures in the air gap may be related to the forces exerted on the supported frame. For the purposes of this series of tests, the desired model was attached to the frame supported by the air bearings and positioned within the magnetic suspension system at the same location and orientation as had been used for the first calibration procedure. Then currents in the magnets were varied independently and recorded. Also, the pressures in the air gaps were recorded and related to the forces and moments the model experienced. The calibration constants for lift, drag, and pitch (A, C and F resp.) were determined from this information and compared with the corresponding constants determined using the first procedure. The constants determined by each procedure were different by 0.3% for drag, 1.0% for lift, and 0.6% for pitch. Thus, both methods yield good correspondence, although the latter method will probably be used more extensively in the future as it represents a substantial simplification when a five (or six) degree of freedom calibration is required.

CHAPTER 4

ALIGNMENT OF THE BALANCE AXIS WITH THE TUNNEL AXIS

Pitch and Yaw

Pitching and yawing moments are produced by the vertical and horizontal field components H_z and H_y respectively. These field components are produced by superposition of orthogonal field components oriented at 45° to the vertical, and orthogonal to the x-axis of the balance. These 45° field components are produced by two independent pairs of coils ("inner and outer saddle coils"). The contribution from each coil current is summed electrically to provide two independent signals, one proportional to the vertical field component, H_z, and one proportional to the horizontal field H_y. These signals are called "I_p", (quasi-pitch current, proportional to H_z) and "I_y" (quasi-yaw current, proportional to H_y).

i.e., $I_p = C_1 I_{is} + C_2 I_{os}$ (4.1)

$$I_y = C_3 I_{is} - C_4 I_{os} \qquad (4.2)$$

where

I_p = pitch current
I_y = yaw current
I_{is} = inner saddle coil current
I_{os} = outer saddle coil current
C_1, C_2, C_3, C_4 = constants

This addition and subtraction is done by two potentiometers which vary the ratios C_1/C_2 and C_3/C_4 independently (see Fig. 6). Therefore, the resulting signals are proportional to the pitch or yaw current. This proportionality is automatically taken into account in the force, moment and current relations defined in Chapter 2.

The procedure for aligning the balance axis with another coordinate system (tunnel geometry coordinate system) is as follows:

A. Alignment in the yaw plane.*

1. Establish 0.0° angle of yaw in the system to which the balance is to be aligned.
2. Suspend model magnetically at this 0.0° angle of yaw in the desired system.
3. Adjust potentiometer for yaw (see Fig. 6) until no interaction of pitch angle with yaw current is experienced.
4. Any direct current offset from zero in the yaw current corresponds to the misalignment between the balance axis and the desired coordinate system.
5. Rotate the magnetic balance in the yaw plane and repeat the procedure until misalignment is within acceptable limits.

The amount of direct current offset can be calibrated as current per yaw angle misalignment, to facilitate the procedure.

B. Alignment in the pitch plane.*

1. Suspend an axisymmetric ferromagnetic body with a fineness ratio (body length/body diameter) of approximately 10:1 by a string at the body's center of gravity. Turn the magnetizing field and the inner and outer saddle coil fields on.
2. Yaw the model while maintaining zero pitch angle with respect to the desired frame of reference.
3. Adjust the potentiometer for pitch (see Fig. 6) current until there is no interaction of pitch current with yaw angle.

*Alignment of the coordinate systems to within ± 0.1° is relatively easy.

4. Any direct current offset from zero in the pitch current corresponds to the misalignment of the balance axis and the desired coordinate system in the pitch plane.
5. Rotate the magnetic balance in the pitch plane and repeat the procedure until the misalignments are within acceptable limits.

Once again the direct current offset can be calibrated as current vs. pitch angle misalignment, to facilitate the procedure.

The reason the model is suspended by a string for the pitch alignment is that the lift current may add to the pitch current as described in Chapter 2, therefore giving a spurious offset. It is not necessary for the yaw plane since the side force current is approximately zero for zero applied sideforce. The lift current, however, is large due to the model weight.

CHAPTER 5

A DYNAMIC DATA ACQUISITION PROCEDURE

The following is the development of an approach for determining stability derivatives using a magnetic suspension system. At present, this procedure is untested, but offers several advancements over procedures previously used. The development assumes small angles of attack and small variations in angle of attack, but will include the effect of rotation about points other than the center of gravity. The frame of reference implied in the development is the body axis with the origin at the center of gravity of the model (see Fig. (7)).

Aerodynamic Force and Moment Equations

The general equations for the pitching moment and lift for as derived in Ref. (5) are resp.*

$$- \frac{\partial M}{\partial \dot{w}} \dot{w} - \frac{\partial M}{\partial w} w + B'\ddot{\theta} - \frac{\partial M}{\partial q} \dot{\theta} = P_M \tag{5.1}$$

and

$$m\ddot{c} - \frac{\partial Z}{\partial \dot{w}} \dot{w} - \frac{\partial Z}{\partial w} w - \frac{\partial Z}{\partial q} \dot{\theta} = F_z \tag{5.2}$$

where

$$\dot{w},\ \dot{\theta},\ \ddot{\theta},\ \text{etc.} = \frac{\partial w}{\partial t}, \frac{\partial \theta}{\partial t}, \frac{\partial^2 \theta}{\partial t^2}, \text{etc.}$$

---

*Static forces and moments are not considered.

m = mass of the model

B' = moment of inertia about the y axis

w = wind velocity along c (see Fig. (7))

u = wind velocity along a (see Fig. (7))

Z = lift force (in c direction)

M = pitching moment (aerodynamic)

F_z = lift force (magnetic)

P_M = pitching moment (magnetic)

$q = \dot{\theta}$

θ = pitch angle

The effective wind velocity (w,u) is affected by the point of rotation in the following manner. First, the displacement of the model in the c-direction is

$$c \sim d\theta + z \tag{5.3}$$

so transforming to the body axis yields,

$$w = u_\infty \sin\theta + d\dot{\theta}\cos\theta \sim u_\infty\theta + d\dot{\theta} \tag{5.4a}$$

$$u = u_\infty \cos\theta - d\dot{\theta}\sin\theta \sim u_\infty \tag{5.4b}$$

where, z = displacement along Z-axis (origin chosen such that $\dot{z} \equiv 0$, i.e., the origin is at the center of rotation)

d = displacement of center of rotation and center of gravity

u_∞ = tunnel velocity (in x-direction)

Using these expressions for u and w, eqns. (5.1) and (5.2) become

$$-\frac{\partial M}{\partial \dot{w}}(u_\infty\dot{\theta} + d\ddot{\theta}) - \frac{\partial M}{\partial w}(u_\infty\theta + d\dot{\theta}) + B'\ddot{\theta} - \frac{\partial M}{\partial q}\dot{\theta} = P_M \tag{5.3}$$

and

$$m d\ddot{\theta} - \frac{\partial Z}{\partial \dot{w}}(u_\infty\dot{\theta} + d\ddot{\theta}) - \frac{\partial Z}{\partial w}(u_\infty\theta + d\dot{\theta}) - \frac{\partial Z}{\partial q}\dot{\theta} = F_z \qquad (5.4)$$

The equations may be nondimensionalized by dividing the moment equation by $\rho u_\infty{}^2 S\ell$ and by dividing the force equation by $\frac{1}{2}\rho u_\infty{}^2 S$ using the conventions of Ref. (5). This yields

$$t^{*2}\ddot{\theta}[i_B - \delta C_{m\dot{\alpha}}] - t^*\dot{\theta}[C_{m\dot{\alpha}} + C_{m_q} + \delta C_{m\alpha}] - \theta[C_{m\alpha}] = \frac{P_m}{\rho u_\infty{}^2 S\ell} \qquad (5.5)$$

and

$$t^{*2}\ddot{\theta}\delta[2\mu - C_{L\dot{\alpha}}] - t^*\dot{\theta}[C_{L\dot{\alpha}} + C_{L_q} + \delta C_{L\alpha}] - \theta[C_{L\alpha}] = \frac{F_z}{\frac{1}{2}\rho u_\infty{}^2 S} \qquad (5.6)$$

where

$\alpha = \tan^{-1}\frac{w}{u_\infty}$

ρ = density of air

S = characteristic area

ℓ = characteristic length

$t^* = \ell/u_\infty$

$\delta = d/\ell$

$\mu = m/\rho S\ell$

$C_{L_{\dot{\alpha}}} = (\frac{\partial Z}{\partial \dot{w}})/\frac{1}{2}\rho_\infty S\ell$

$C_{L_q} = (\frac{\partial Z}{\partial q})/\frac{1}{2}\rho u_\infty S\ell$

$C_{L\alpha} = (\frac{\partial Z}{\partial w})/\frac{1}{2}\rho u_\infty S$

$i_B = B'/\rho S\ell^3$

$C_{m\dot{\alpha}} = (\frac{\partial M}{\partial \dot{w}})/\rho S\ell^2$

$C_{m_q} = (\frac{\partial M}{\partial q})/\rho u_\infty S\ell^2$

$C_{m\alpha} = (\frac{\partial M}{\partial w})/\rho u_\infty S\ell$

To first order P_M and F_z are related to the magnet currents according to the following relation (see Chapter 2)

$$P_M = F\, I_x\, I_p - h \cdot C\, I_x I_L \tag{5.7}$$

and

$$F_z = C\, I_x I_L \tag{5.8}$$

where,

C, F = constants of proportionality

h = separation of center of gravity and magnetic moment center

I_x = magnetizing current (constant)

I_L = lift current

I_p = pitch current

thus eqns. (5.5) and (5.6) become

$$t^{*2}\ddot{\theta}[i_B - \delta C_{m\dot{\alpha}}] - t^*\dot{\theta}[C_{m_{\dot{\alpha}}} + C_{m_q} + \delta C_{m\alpha}] - \theta[C_{m\alpha}] = \frac{F\, I_x I_p - h \cdot C I_x I_L}{\rho u_\infty^2\, S\ell} \tag{5.9}$$

and

$$t^{*2}\ddot{\theta}\delta[2\mu - C_{L\dot{\alpha}}] - t^*\dot{\theta}[C_{L\dot{\alpha}} + C_{L_q} + \delta C_{L\alpha}] - \theta[C_{L\alpha}] = \frac{C I_x\, I_L}{\frac{1}{2}\rho u_\infty^2 S} \tag{5.10}$$

If now, the two preceding equations are Laplace transformed to allow algebraic manipulation the result is

$$t^{*2}[i_B - \delta C_{m\dot{\alpha}}]s^2\theta - t^*[C_{m\dot{\alpha}} + C_{m_q} + \delta C_{m\alpha}]s\theta - [C_{m\alpha}]\theta = C_2 I_p - \frac{\varepsilon}{2} C_1 I_L \tag{5.11}$$

and

$$t^{*2}\delta[2\mu - C_{L\dot{\alpha}}]s^2\theta - t^*[C_{L\dot{\alpha}} + C_{L_q} + \delta C_{L\alpha}]\, s\theta - [C_{L\alpha}]\theta = C_1 I_L \tag{5.12}$$

where,

$$\theta = \theta(s)$$
$$I_L = I_L(s)$$
$$I_p = I_p(s)$$
$$s = \text{Laplace variable}$$
$$c_1 = \frac{CI_x}{\rho u_\infty^2 S/2}$$
$$c_2 = \frac{FI_x}{\rho u_\infty^2 S\ell}$$
$$\varepsilon = h/\ell$$

If eqn. (5.11) is solved for I_L and the result substituted for I_L in eqn. (5.12), the result is

$$t^{*2}[i_B - \delta C_{m\dot{\alpha}}]\, s^2\theta - t^*[C_{m\dot{\alpha}} + C_{m_q} + \delta C_{m\alpha}]\, s\theta - [C_{m\alpha}]\theta =$$
$$C_2 I_p - \frac{\varepsilon}{2}\{t^{*2}\delta[2\mu - C_{L\dot{\alpha}}]s^2\theta - t^*[C_{L\dot{\alpha}} + C_{L_q} + \delta C_{L\alpha}]s\theta - [C_{L\alpha}]\theta\} \tag{5.13}$$

or

$$t^{*2}[i_B - \delta C_{m\dot{\alpha}} + \frac{\varepsilon}{2}\delta\{2\mu - C_{L\dot{\alpha}}\}]s^2\theta - t^*[C_{m\dot{\alpha}} + C_{m_q} + \delta C_{m\alpha} +$$
$$\frac{\varepsilon}{2}\{C_{L\dot{\alpha}} + C_{L_q} + \delta C_{L\alpha}\}]s\theta - [C_{m\alpha} + \frac{\varepsilon}{2}C_{L\alpha}]\,\theta = C_2 I_p \tag{5.14}$$

In addition to the aerodynamic damping and stiffness, there exists damping and stiffness due to the magnetic suspension system. These effects may be incorporated as follows:

$$t^{*2}[i_B\ldots]s^2\theta - t^*[(C_{m\dot{\alpha}}+\ldots+M_m]s\theta - [(C_{m\alpha}+ \frac{\varepsilon}{2} C_{L\alpha}+K_M]\theta = C_2 I_p \tag{5.15}$$

and

$$t^{*2}\delta[2\mu-C_{L\dot{\alpha}}]s^2\theta - t^*[C_{L\dot{\alpha}}+C_{L_q}+\delta C_{L\alpha}+M'_m]s\theta - [C_{L\alpha}+K'_m]\theta = C_1 I_L \tag{5.16}$$

where K_m is proportional to magnetic stiffness in pitch

M_m is proportional to magnetic damping in pitch

K'_m is proportional to magnetic stiffness in lift

M'_m is proportional to magnetic damping in lift

Wind off equations, (5.15) and (5.16), become simply,

$$t^{*2}[i_B + \delta\varepsilon\mu]s^2\theta - t^* M_m s\theta - K_m\theta = C_2 I_p \tag{5.17}$$

and

$$2t^{*2}\delta\mu s^2\theta - t^* M'_m s\theta - K'_m\theta = C_1 I_L \tag{5.18}$$

To obtain solutions for the stability derivatives in eqns. (5.15) and (5.16), the coefficients of the second order terms must be known. But, since it is known that

$$i_B >> \delta C_{m_{\dot{\alpha}}} \tag{5.19}$$

and

$$2\mu >> C_{L_{\dot{\alpha}}} \tag{5.20}$$

eqns (5.15) and (5.16) may be approximated by

$$t^{*2}[i_B + \varepsilon\delta\mu]\ s^2\theta - \ldots\ldots = C_2 I_p \tag{5.21}$$

and

$$t^{*2}[2\delta\mu]\ s^2\theta - \ldots\ldots = C_l I_L \qquad (5.22)$$

Eqns. (5.17), (5.18), (5.21), and (5.22) are of the form

$$s^2\theta + 2\zeta\omega_o s\theta + \omega_o^{\,2}\theta = K\,\overline{m_f} \qquad (5.23)$$

where,

ζ = damping ratio

ω_o = natural frequency

K = constant

$\overline{m_f}$ = Laplace transform of the forcing function

There are several methods of solving for the damping ratio and natural frequency (The forced oscillation technique and the phase shift methods are described in Ref. (3)). Therefore, only the final results are shown below:

$$C_{L_\alpha} = \frac{dm}{qs}\,[\omega'^{\,2}_{o_L} - \omega_{o_L}^{\,2}] \qquad (5.24)$$

$$C_{L_{\dot\alpha}} + C_{L_q} = -\frac{d}{\ell}\,C_{L_\alpha} + \frac{2dmu_\infty}{qS\ell}\,[\zeta'_L\,\omega'_{o_L} - \zeta_L\omega_{o_L}] \qquad (5.25)$$

$$C_{m_\alpha} = -\frac{h}{2\ell}\,C_{L_\alpha} + \frac{1}{2qS\ell}\,(B' + mhd)\,[\omega'^{\,2}_{o_p} - \omega_{o_p}^{\,2}] \qquad (5.26)$$

$$C_{m_{\dot\alpha}} + C_{m_q} = -\frac{d}{\ell}\,C_{m\alpha} - \frac{h}{2\ell}\,(C_{L_{\dot\alpha}} + C_{L_q} + \frac{d}{\ell}\,C_{L_\alpha}) + \frac{u_\infty}{qS\ell^2}\,(B' + mhd)\,[\zeta'_p\,\omega'_{o_p} - \zeta_p\,\omega_{o_p}] \qquad (5.27)$$

where

where

ω_{o_L} = wind-on lift natural frequency

ω_{o_p} = wind-on pitch natural frequency

ζ_{o_L} = wind-on lift damping

ζ_{o_p} = wind-on pitch damping

ω'_{o_L} = wind-off lift natural frequency

ω'_{o_p} = wind-off pitch natural frequency

ζ'_{o_L} = wind-off lift damping

ζ'_{o_p} = wind-off pitch damping

Center of rotation determination

The separation of the center of rotation and the center of gravity must also be known to solve eqns. (5.15) and (5.16). This may be determined during the wind tunnel run by evaluating the amplitude ratio of the lift and pitch position signals from the electromagnetic position sensor. In explanation, the following development is presented as justification.

The lift and pitch position signals are proportional to lift position and pitch angle at a point not necessarily at the center of gravity,

i.e.,

$$z = K_1 P_L + K_2 + b'\theta \tag{5.28}$$

$$\theta = K_3 P_p + K_4 \tag{5.29}$$

where

K_1, K_2, K_3, K_4 = constants

b' = separation of position sensor center and center of gravity

P_L = lift position signal

P_p = pitch position signal

combining eqns. (5.3) and (5.28) yields

$$d\theta = K_1 P_L + K_2 + b\theta \tag{5.30}$$

combining this result and eqn. (5.29) yields

$$d - b' = \frac{K_1 P_L + K_2}{K_3 P_p + K_4} \tag{5.31}$$

Now, if P_L and P_p are assumed to be sine waves at the driving frequency, i.e.,

$$P_L = \overline{P_L} \sin(\omega t + \gamma) \tag{5.32}$$

$$P_p = \overline{P_p} \sin(\omega t + \beta) \tag{5.33}$$

where

$\overline{P_L}, \overline{P_p}$ = constants

β, γ = phase angles

ω = driving frequency

then eqn. (5.31) can be written as

$$(d-b')K_3\overline{P_p}\,\sin(\omega t + \beta) - K_1\overline{P_L}\,\sin(\omega t + \gamma) = K_2 - K_4(d-b') \quad (5.34)$$

for eqn. (5.34) to be true, both sides must equal zero which implies

$$d-b' = \frac{K_1\overline{P_L}\,\sin(\omega t + \gamma)}{K_3\overline{P_p}\,\sin(\omega t + \beta)} \quad (5.35)$$

for eqn. (5.35) to be true the phase angle must be equal ($\gamma=\beta$), implying

$$d-b' = \frac{K_1\overline{P_L}}{K_3\overline{P_p}} \quad (5.36)$$

In practice, this ratio may be determined using the same system used to determine the amplitude ratios ($\left|\frac{\theta}{I_L}\right|$, $\left|\frac{\theta}{I_p}\right|$) in the preceding section. This system yields a voltage proportional to the log of the amplitude ratio of the input signals. In this case,

$$V' = K_7 \log \frac{\overline{P_L}}{\overline{P_p}} + K_8 \quad (5.37)$$

where,

V' = the voltage from the amplitude ratio measurement system

K_7, K_8 = constants

so, the final result is

$$\log\ (d-b') = \frac{1}{K_7}\ (V' - K_8) + \log \frac{K_1}{K_3} \tag{5.38}$$

Therefore, a particular value of V' will correspond to a particular d (or point of rotation). The point of rotation may be varied until the desired V' corresponding to the desired point of rotation is attained.

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3. Vlajinac, M. and Gilliam, G. D., "Aerodynamic Testing on Conical Configurations using a Magnetic Suspension System," ARL 70-0067, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, April 1970.

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5. Etkin, B., Dynamics of Flight, John Wiley and Sons, Inc., 1959.

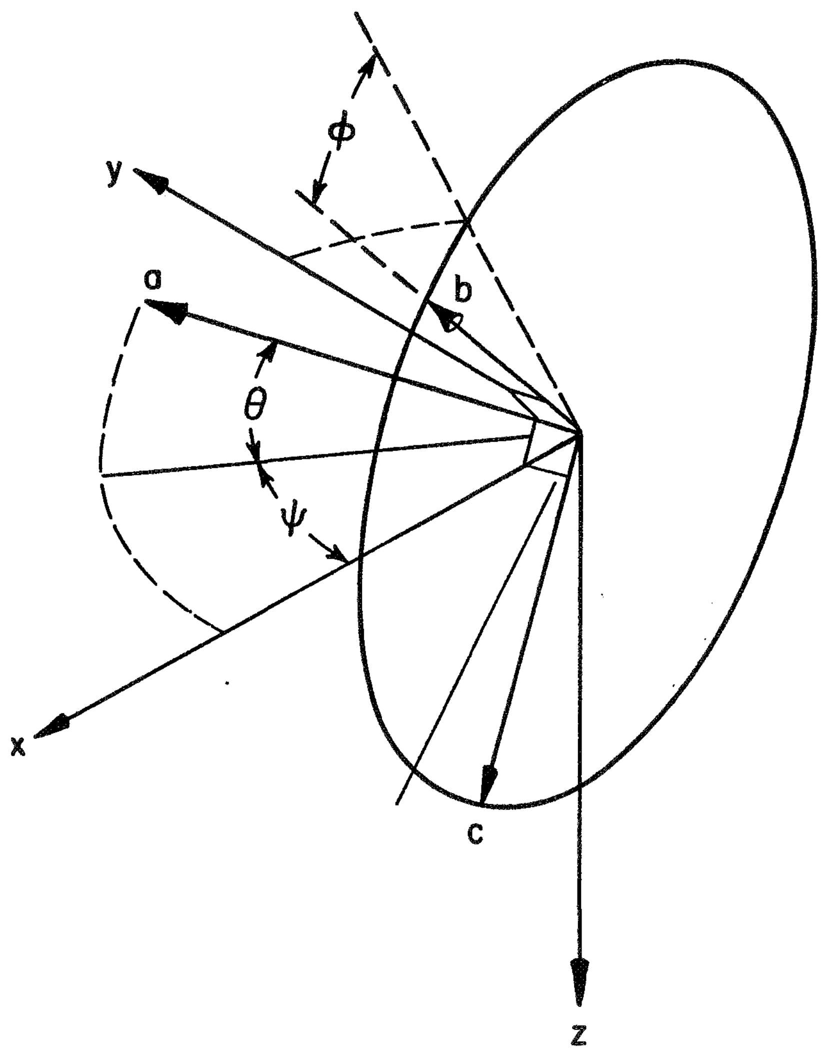


Figure 1. MODEL AND WIND TUNNEL AXIS SYSTEM

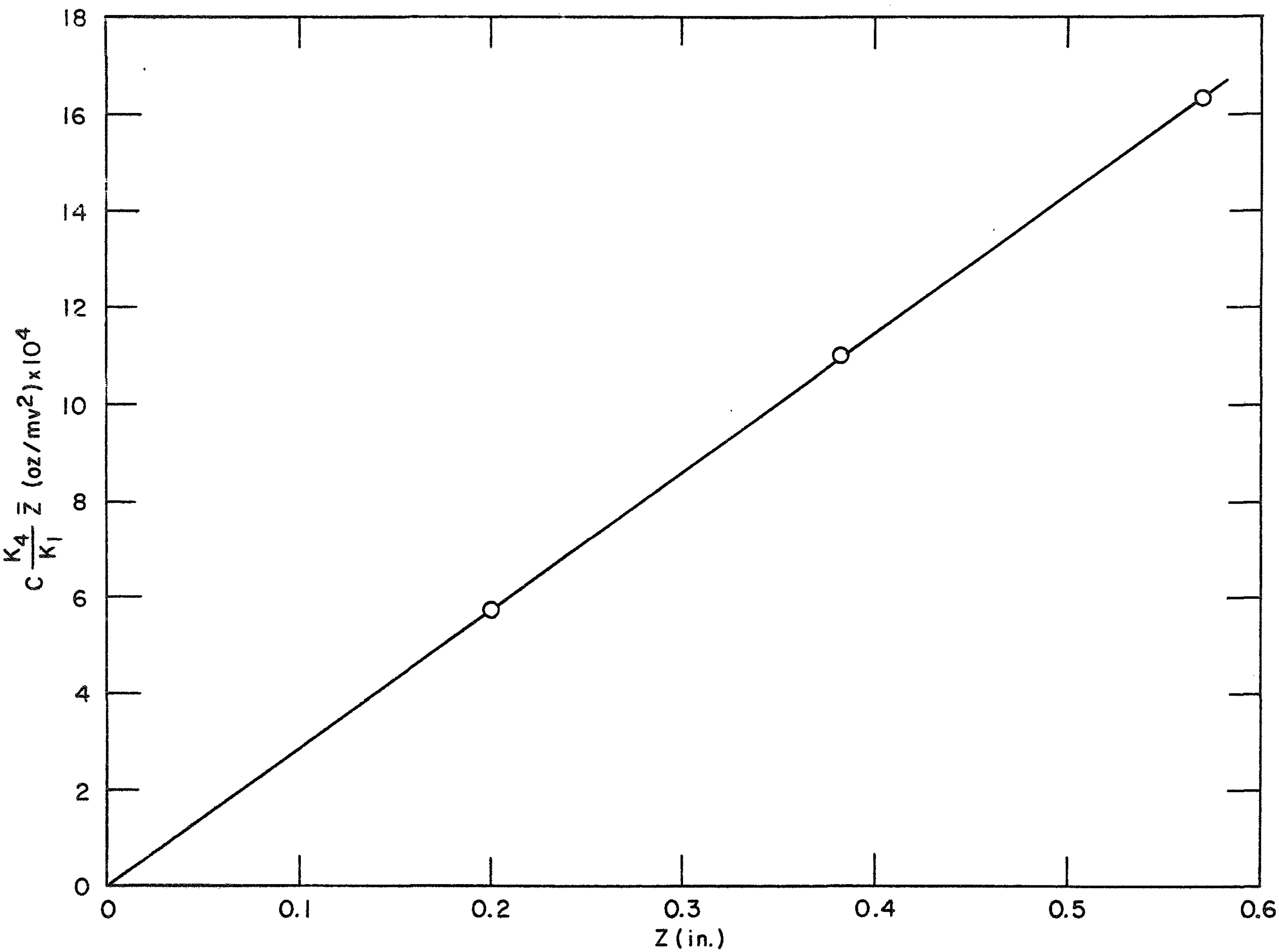


Figure 2. VARIATION OF THE CORRECTION TO THE MAGNETIZING CURRENT DUE TO LIFT CURRENT WITH LIFT POSITION

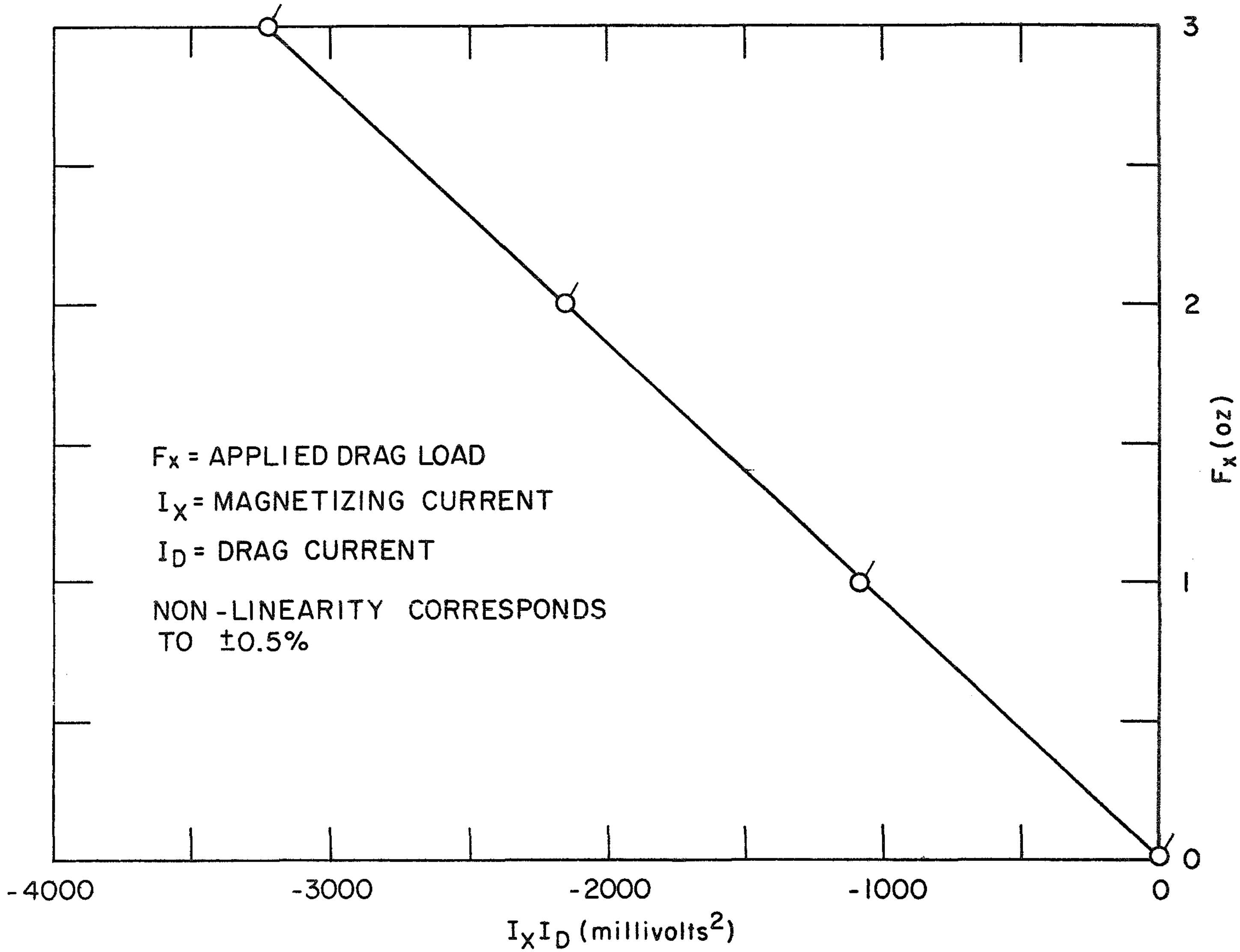


Figure 3. DRAG FORCE VERSUS THE PRODUCT OF THE MAGNETIZING AND DRAG CURRENT ($I_x I_D$)

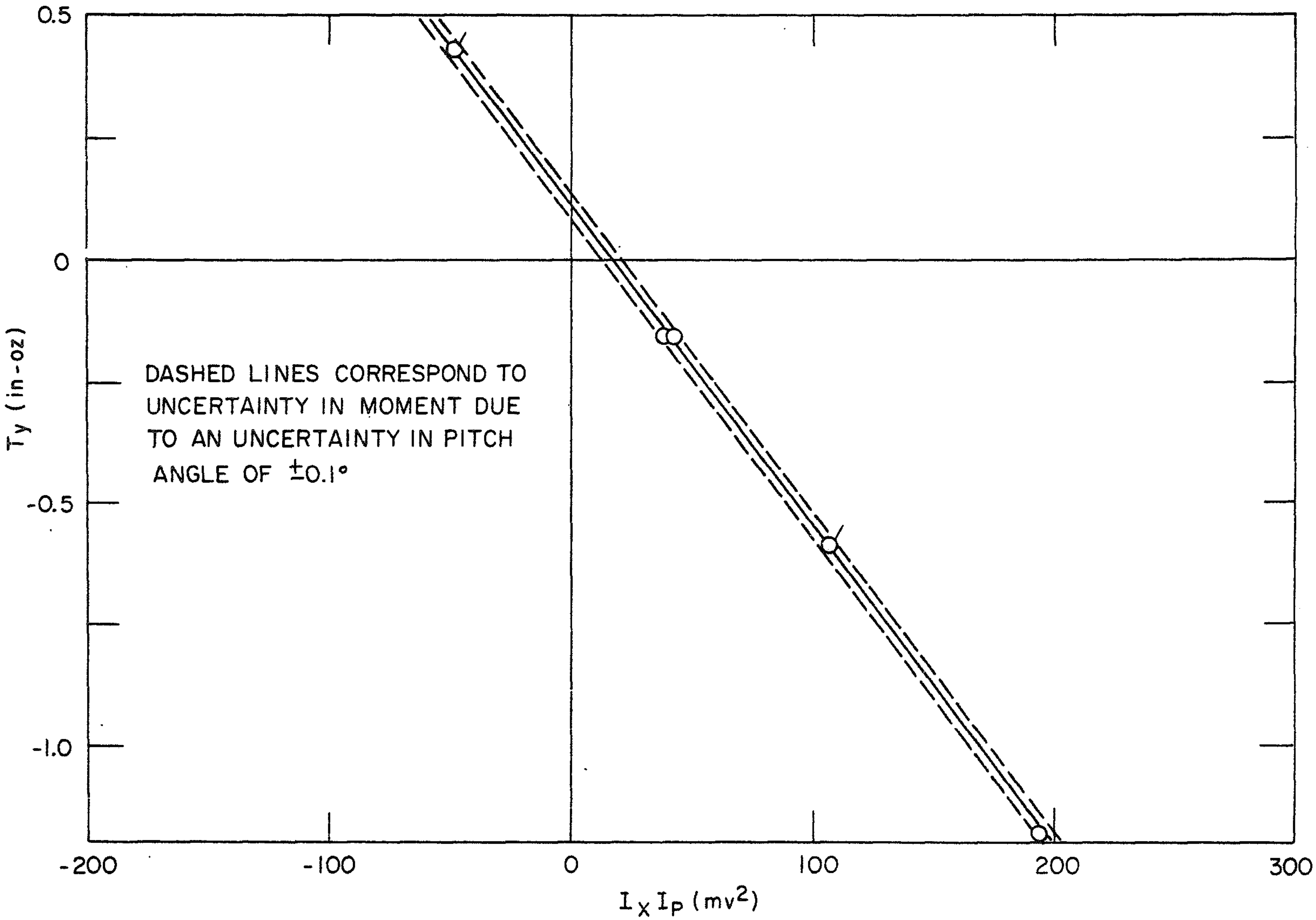


Figure 4. APPLIED TORQUE VERSUS THE PRODUCT OF THE MAGNETIZING AND PITCH CURRENTS FOR A DELTA WING CONFIGURATION

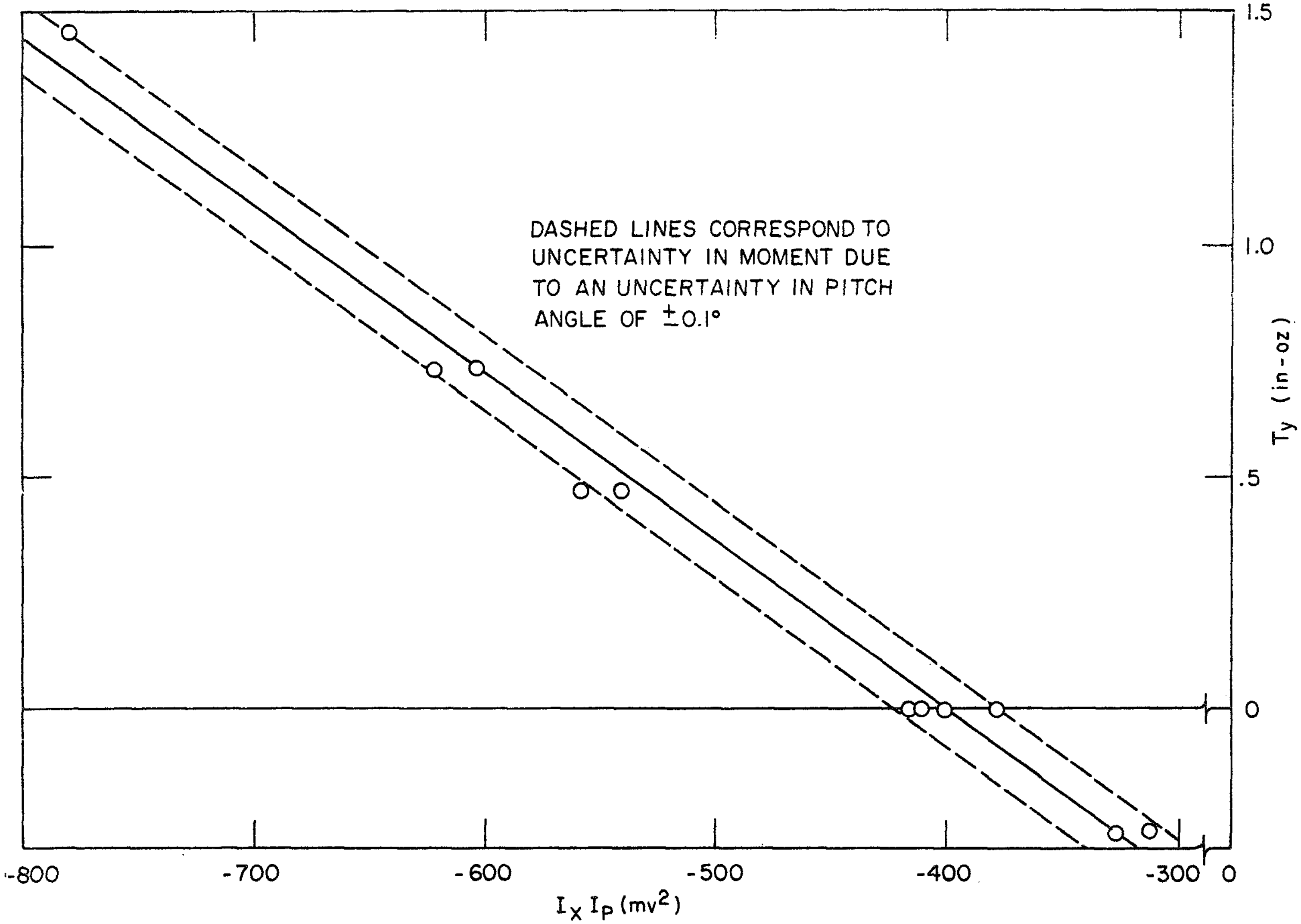
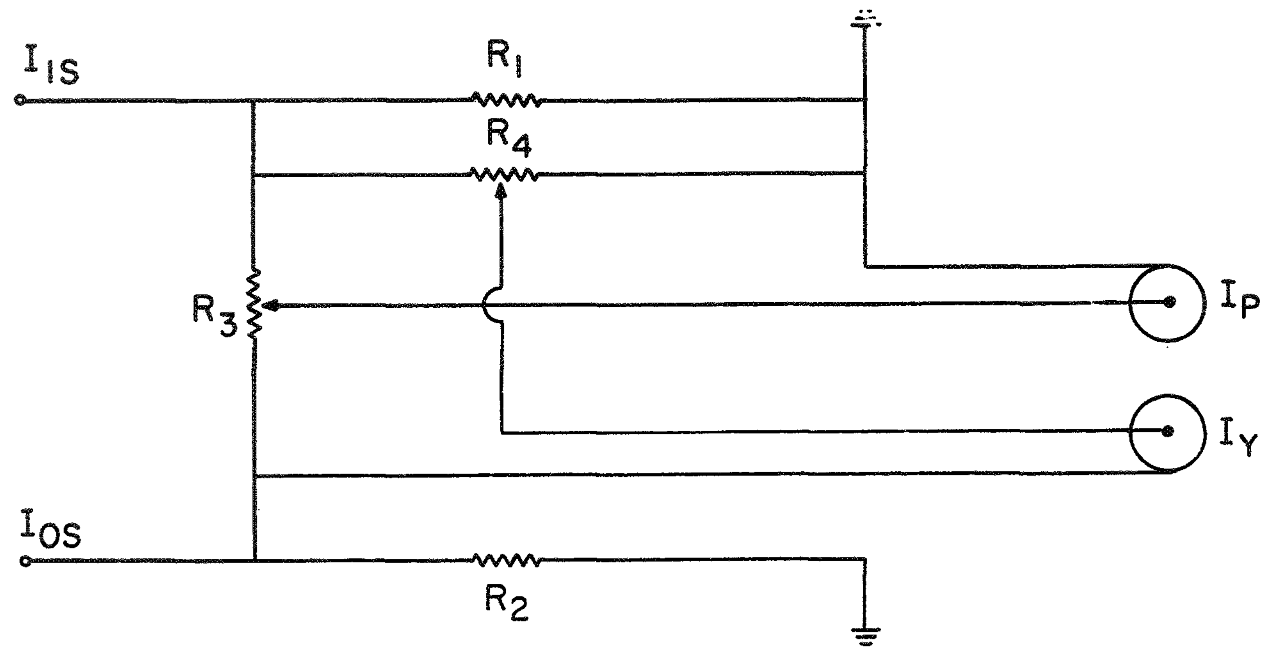


Figure 5. APPLIED TORQUE VERSUS THE PRODUCT OF THE MAGNETIZING AND PITCH CURRENTS FOR A 6° HALF ANGLE CONE

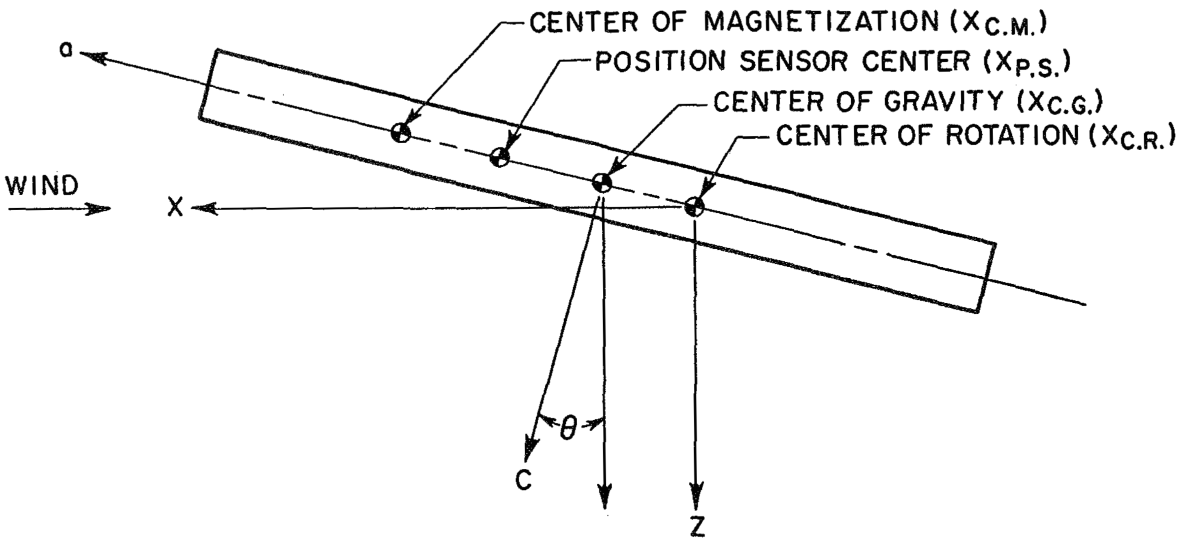


R_1, R_2 = SHUNT RESISTANCES

R_3 = PITCH POTENTIOMETER

R_4 = YAW POTENTIOMETER

Figure 6. SCHEMATIC OF CIRCUIT FOR OBTAINING THE PITCH AND YAW CURRENTS FROM THE INNER AND OUTER SADDLE COIL CURRENTS



d = DISPLACEMENT OF CENTER OF ROTATION FROM THE CENTER OF GRAVITY ($X_{C.R.} - X_{C.G.}$)

h = DISPLACEMENT OF CENTER OF MAGNETIZATION AND THE CENTER OF GRAVITY ($X_{C.M.} - X_{C.G.}$)

b = DISPLACEMENT OF POSITION SENSOR CENTER AND THE CENTER OF GRAVITY ($X_{P.S.} - X_{C.G.}$)

Figure 7. LOCATION OF VARIOUS CENTRAL POINTS REFERENCED TO THE WIND TUNNEL AXIS

APPENDIX A

STATIC DATA REDUCTION COMPUTER PROGRAM FOR USE WITH THE PROTOTYPE MAGNETIC SUSPENSION SYSTEM IN BOTH THE MACH 4.25 OPEN JET TUNNEL AND THE DYNAMIC PRESSURE SIMULATOR AT THE M.I.T. AEROPHYSICS LABORATORY

```
  DIMENSION ALPHA(30), PSI(30), THETA(30), VXO(30), VPO(30), VYO(30)
 1,VLO(30), VDO(30), VSO(30)
1 CONTINUE

      THE NEXT STATEMENT READS THE FIRST DATA CARD.  THE FIRST
 COLUMN SHOULD CONTAIN THE VARIABLE 'JUST', THE SECOND THE
 VARIABLE 'ICORE', AND THE COLUMNS 3-72 A DESCRIPTION
 OF THE TEST IF SUCH A DESCRIPTION IS DESIRED PRINTED
 WITH THE REDUCED DATA.  THE FUNCTION OF THE VARIABLES 'JUST'
 AND 'ICORE' ARE DESCRIBED LATER.

 2 READ(2,1000) JUST, ICORE, (ALPHA(I),I=1,18)
   IF (JUST) 3,3000,3
 3 CONTINUE
 4 WRITE (3,2000) (ALPHA(I),I=1,18)
   IF (1-JUST) 20, 21, 20
21 CONTINUE
   WRITE (3,2001)
   GO TO 22
20 CONTINUE
   WRITE (3,2006)
22 CONTINUE

      LOGICAL DECISION   CONSTANTS DESCRIBING CONDITIONS
 ARE ASSUMED THE SAME AS FOR THE PREVIOUS MODEL IF 'ICORE'
 EQUALS ZERO, OTHERWISE THEY WILL BE READ IN AGAIN.

   IF (ICORE) 5,18,5

      THESE ARE THE CONSTANTS USED IN THE DATA REDUCTION.
 THEY ARE DEFINED AS FOLLOWS

      A,B,C,D,E,F,G= MAGNET CONSTANTS DEFINED PREVIOUSLY
      DADB=DA/DB=RATIO OF DEMAGNETIZING FACTORS
      C2C1X=(A*G/F)*((DA/DB)-1)*X= CORRECTION FACTOR DUE TO DISPLACEMENT
      C2C1K = THIRD ORDER CORRECTION TO MAGNETIZING CURRENT
      C4C1Z=(C*G/F)*((DA/DB)-1)*Z= CORRECTION FACTOR DUE TO DISPLACEMENT
      AREA = REFERENCE AREA FOR NON-DIMENSIONALIZATION
      DIMEN = REFERENCE LENGTH FOR NON-DIMENSIONALIZATION
      XREF = REFERENCE LENGTH FOR CENTER OF PRESSURE CALCULATION
      XCM = LOCATION OF CENTER OF MAGNETIZATION FROM NOSE OF MODEL
      XMOM = MOMENT REFERENCE POINT FROM NOSE
      DS = PRESSURE CALIBRATION CONSTANT FOR SUBSONIC TUNNEL
      DA,DB,DE,DC,DG,DF,DO,DP = TUNNEL CORRECTION CONSTANTS, SEE
           EQUATIONS PRECEDING STATEMENT 1000 FOR INTERPRETATION.

 5 READ (2,1002) A,B,C,D,E,F,G,DADB,C2C1X,C2C1K,C4C1Z
   READ (2,1001) AREA,DIMEN,XREF,XCM,XMOM,DS,DA,DB,DE,DC,DG,DF,DO,DP
18 J=1
```

```
6 CONTINUE

C      THE NEXT TWO READ STATEMENTS READ IN TARE DATA.  EVERY
C  ANGLE OF ATTACK TESTED MUST HAVE TARE DATA CORRESPONDING TO THAT
C  ANGLE OF ATTACK.  THE VARIABLES ARE DEFINED AS FOLLOWS

C       KAT = BRANCH CONSTANT    IF ZERO, NO MORE TARES WILL BE
C             ASSUMED.  IF NON-ZERO, ASSUMES ADDITIONAL TARES
C             FOLLOW.
C       THETA(J) = PITCH ANGLE
C       PSI(J) = YAW ANGLE
C       VLO(J), VYO(J), VPO(J), VDO(J), VSO(J), AND VXO(J)
C             CORRESPOND TO THE WIND OFF VALUES OF IL, IY, IP, ID,
C             IS, AND IX RESP. AT THE ORIENTATION PREVIOUSLY
C             DESCRIBED.

      READ (2,1003) KAT,THETA(J),PSI(J)
      IF (KAT) 7,8,7
    7 READ (2,1004) VLO(J), VYO(J), VPO(J), VDO(J), VSO(J), VXO(J)
      VXO(J)=VXO(J)+C2C1X*VDO(J)+C2C1K*(VDO(J)*VDO(J)/VXO(J))+C4C1Z*VLO(
     1J)
      JMAX=J
      J=J+1
      GO TO 6

C      THE NEXT TWO READ STATEMENTS READ IN THE WIND ON
C  CURRENTS, AND TUNNEL CONDITIONS.  THE VARIABLES ARE AS
C  FOLLOWS     (SUBSONIC MEANING IN PARENTHESIS)

C       IRUN1, IRUN, AND IRUN2 CORRESPOND TO A RUN NUMBER.  IF 'IRUN'
C             EQUALS ZERO ASSUMES DATA READING IS COMPLETE AND
C             RETURNS TO STATEMENT 1. (SAME)
C       TO = TOTAL TEMPERATURE ( ROOM TEMPERATURE )
C       PO = TOTAL PRESSURE ( ROOM PRESSURE )
C       PSI1 = YAW ANGLE (SAME)
C       THET1 = PITCH ANGLE (SAME)
C       PSET = ( PRESSURE SET RATIO TO DYNAMIC PRESSURE )
C             NO MEANING FOR SUPERSONIC DATA
C       VL, VY, VP, VD, VS, AND VX EQUAL MAGNET CURRENTS IL, IY,
C             IP, ID, IS, AND IX RESP..

    8 READ (2,1005) IRUN1,IRUN,IRUN2,TO, PO,PSI1,THET1 ,PSET
      IF (IRUN)9,1,9
    9 READ (2,1006) VL, VY, VP, VD, VS, VX
      J=1
   13 CONTINUE
      IF (THETA(J)-THET1)  11,10,11
   10 IF (PSI(J)-PSI1) 11,14,11
   11 IF (JMAX-J) 3000,3000,12
   12 J=J+1
      GO TO 13
   14 CONTINUE

C      THE FOLLOWING ARE THE DATA REDUCTION EQUATIONS TO YIELD
C  THE AERODYNAMIC COEFFICIENTS.

      TSIN=SIN(THET1 *1.7453E-02)
      TCOS=COS(THET1 *1.7453E-02)
      PSIN=SIN(PSI1*1.7453E-02)
      PCOS=COS(PSI1*1.7453E-02)
```

```
      DAB=1.0-DADB
      VX=VX+C2C1X*VD+C2C1K*(VD*VD/VX)+C4C1Z*VL
      VDVX=VD*VX-VDO(J)*VXO(J)
      VDVY=VD*VY-VDO(J)*VYO(J)
      VDVP=VD*VP-VDO(J)*VPO(J)
      VSVX=VS*VX-VSO(J)*VXO(J)
      VSVY=VS*VY-VSO(J)*VYO(J)
      VSVP=VS*VP-VSO(J)*VPO(J)
      VLVX=VL*VX-VLO(J)*VXO(J)
      VLVY=VL*VY-VLO(J)*VYO(J)
      VLVP=VL*VP-VLO(J)*VPO(J)
      VXVY=VX*VY-VXO(J)*VYO(J)
      VXVX=VX*VX-VXO(J)*VXO(J)
      VYVY=VY*VY-VYO(J)*VYO(J)
      VPVY=VP*VY-VPO(J)*VYO(J)
      VXVP=VX*VP-VXO(J)*VPO(J)
      VPVP=VP*VP-VPO(J)*VPO(J)
      TSIN2=TSIN**2
      TCOS2=TCOS**2
      PSIN2=PSIN**2
      PCOS2=PCOS**2

           FORCE - CURRENT RELATIONS

      FX=A*VDVX*(DADB+DAB*TCOS2*PCOS2) + A*E*VDVY*DAB*TCOS2*PSIN*PCOS
     1 -A*G*VDVP*DAB*TSIN*TCOS*PCOS + B*VSVX*DAB*TCOS2*PSIN*PCOS
     2 +B*E*VSVY*(DADB+DAB*PSIN2*TCOS2) - B*G*VSVP*DAB*PSIN*TSIN*TCOS
     3 -C*VLVX*DAB*TSIN*TCOS*PCOS   - C*E*VLVY*DAB*TSIN*TCOS*PSIN
     4 +C*G*VLVP*(DADB+DAB*TSIN2)

      FY=B*VSVX*(DADB+DAB*TCOS2*PCOS2) + B*E*VSVY*DAB*TCOS2*PSIN*PCOS
     1 -B*G*VSVP*DAB*TSIN*TCOS*PCOS - 0.5*A*VDVX*DAB*TCOS2*PSIN*PCOS
     2 -0.5*A*E*VDVY*(DADB+DAB*PSIN2*TCOS2)
     3 + 0.5*A*G*VDVP*DAB*TSIN*TCOS*PSIN

      FZ=C*VLVX*(DADB+DAB*TCOS2*PCOS2) + C*E*VLVY*DAB*TCOS2*PSIN*PCOS
     1 -C*G*VLVP*DAB*PCOS*TSIN*TCOS + 0.5*A*VDVX*DAB*PCOS*TSIN*TCOS
     2 +0.5*A*E*VDVY*DAB*PSIN*TSIN*TCOS
     3 -0.5*A*G*VDVP*(DADB+DAB*TSIN2)

           MOMENT - CURRENT RELATIONS

      TY=F*VXVP*(TCOS2*PCOS2-TSIN2)
     1    + ((F/G)*VXVX-F*G*VPVP)*TSIN*TCOS*PCOS
     2    + F*E*VPVY*TCOS2*PSIN*PCOS
     3    + D*VXVY*TSIN*PSIN*TCOS

      TZ=D*VXVY*(PSIN2*TCOS2-TCOS2*PCOS2)
     1    + ((D/E)*VXVX-D*E*VYVY)*TCOS2*PSIN*PCOS
     2    + D*G*VPVY*TSIN*TCOS*PCOS
     3    - F*VXVP*TSIN*PSIN*TCOS


     BRANCHES ON 'JUST', FOR THE FOLLOWING CONDITIONS

           (1) 'JUST' EQUALS ZERO IMPLIES NO MORE DATA.
           (2) 'JUST' EQUAL TO ONE IMPLIES SUPERSONIC DATA
           (3) 'JUST' EQUAL TO TWO IMPLIES SUBSONIC DATA

      IF (1-JUST) 16,15,16
```

```
  15 CONTINUE
     REL=(PO/12.0)*((1.6848E+7*TO)+2.3183E+10)/((TO   +459.69)**2)
    1 *DIMEN
     Q=PO*0.05999
     CD=FX/(Q*AREA*16.0)
     CL=FZ/(Q*AREA*16.0)
     CS=FY/(Q*AREA*16.0)
     PM=TY/(Q*AREA*DIMEN*16.0)
     YM=TZ/(Q*AREA*DIMEN*16.0)
     XCP=((XCM-XREF)/DIMEN)-(PM/(CL*TCOS+CD*TSIN))
     PM=PM+((XMOM-XCM)/DIMEN)*(CL*TCOS+CD*TSIN)
     WRITE (3,2002) IRUN1,IRUN,IRUN2,TO,PO,Q,REL,THET1,PSI1,CD,CL,CS,YM
    1,PM
     WRITE (3,2003) XCP,FX,FZ,FY,TZ,TY
     GO TO 8
  16 CONTINUE
     SIGMA=(PO/29.92)*(288.0/(TO+273.0))
     PSET=DS*PSET
     C1=0.030257-21.4E-06*TO
     PSET=C1*PSET
     Q=7.232*PSET+4.3257*PSET*PSET
     V=348.07*SQRT(Q/SIGMA)
     S=V/SQRT(4324.32*TO+1.42126E+06)
     REL=(DIMEN/12.0)*(SIGMA*V*2.378E+06)/(358.3+0.987*TO)
     CD=FX/(Q*AREA*16.0)
     CL=FZ/(Q*AREA*16.0)
     CS=FY/(Q*AREA*16.0)
     PM=TY/(Q*AREA*DIMEN*16.0)
     YM=TZ/(Q*AREA*DIMEN*16.0)
     CD1=CD
     CL1=CL
     CD=(DA-DB*CD1)*CD1+DE*CL1*CL1
     CL=(DC-DG*CD1)*CL1
     PM=(DF-DO*CD1)*PM+DP*CL1
     XCP=((XCM-XREF)/DIMEN)-(PM/(CL*TCOS+CD*TSIN))
     PM=PM+((XMOM-XCM)/DIMEN)*(CL*TCOS+CD*TSIN)
     WRITE (3,2004) IRUN1,IRUN,IRUN2,PSET,S,Q,REL,THET1,PSI1,CD,CL,CS,Y
    1M,PM
     WRITE (3,2005) XCP,FX,FZ,FY,TZ,TY
     GO TO 8
1000 FORMAT(2I1,A2,17A4)
2000 FORMAT (1H1//30X,A2,17A4///)
2001 FORMAT (4X,3HRUN,5X,6HTOTAL ,4X,6HTOTAL ,2X,7HDYNAMIC,2X,8HREYNOLD
    1S,1X,5HPITCH,2X,3HYAW/2X,6HNUMBER,1X,11HTEMPERATURE,1X,8HPRESSURE,
    21X,8HPRESSURE,2X,6HNUMBER,2X,5HANGLE,1X,5HANGLE,5X,4HDRAG,8X,4HLIF
    3T,8X,4HSLIP,8X,4HYAW ,7X,5HPITCH//)
1001 FORMAT (6F10.3/4F10.3/4F10.3)
1002 FORMAT (4E15.3/4E15.3/3E15.3)
1003 FORMAT (I1,F9.3,F10.3)
1004 FORMAT (6F10.3)
1005 FORMAT (A3,I3,A1,3X,5F10.3)
1006 FORMAT (6F10.3)
2002 FORMAT(1H ,A3,I3,A1,F8.1,F10.1,F9.4,E12.4,2F5.1,5H  CD=,F8.5,4H CL=,
    2=,F8
    1.5,4H CS=,F8.5,4H CN=,F8.5,4H CM=,F8.5)
2003 FORMAT (1H ,10X,33HCENTER OF PRESSURE IS LOCATED AT ,F7.4,10X,
    1                    4H FX=,F7.3,5H  FZ=,F7.3,5H  FY=,F7.3,5H  TZ=,F7.3
    1,4H TY=,F7.3/)
2004 FORMAT(1H ,A3,I3,A1,F8.3,F10.4,F9.4,E12.4,2F5.1,5H  CD=,F8.5,4H CL=,
    2=,F8
```

```
     1.5,4H CS=,F8.5,4H CN=,F8.5,4H CM=,F8.5)
2005 FORMAT (1H ,10X,33HCENTER OF PRESSURE IS LOCATED AT ,F7.4,10X,
     1                 4H FX=,F7.3,5H  FZ=,F7.3,5H  FY=,F7.3,5H  TZ=,F7.3
     1,4H TY=,F7.3/)
2006 FORMAT (4X,3HRUN,4X,8HPRESSURE,2X,8HMACH NO.,1X,7HDYNAMIC,2X,8HREY
     1NOLDS,1X,5HPITCH,2X,3HYAW/2X,6HNUMBER,5X,3HSET,13X,
     21X,8HPRESSURE,2X,6HNUMBER,2X,5HANGLE,1X,5HANGLE,5X,4HDRAG,8X,4HLIF
     3T,8X,4HSLIP,8X,4HYAW ,7X,5HPITCH//)
3000 CALL EXIT
     END
```

APPENDIX B

NOTES ON COMPUTER OUTPUT

Symbols

CD=C_D	Drag coefficient
DL=C_L	Lift coefficient
CS=C_S	Side force coefficient
CN=C_N	Yawing moment coefficient
CM=C_M	Pitching moment coefficient
FX,FY,FZ,TZ,TY=F_x,F_y,F_z,T_z,T_y	Forces and Moments (oz, in-oz)

RUN NUMBER	PRESSURE SET	MACH NO.	DYNAMIC PRESSURE	REYNOLDS NUMBER	PITCH ANGLE	YAW ANGLE	DRAG	LIFT	SLIP	YAW	PITCH
1-101G	0.005	0.0546	0.0365	0.9098E 05	4.0	0.0	CD= 0.02523	CL= 0.09906	CS= 0.00178	CN=-0.00267	CM=-0.07347
	CENTER OF PRESSURE IS LOCATED AT 0.7304						FX= 0.057	FZ= 0.278	FY= 0.004	TZ= -0.019	TY= -0.235
1-102G	0.005	0.0548	0.0369	0.9141E 05	6.0	0.0	CD= 0.03532	CL= 0.15871	CS=-0.00456	CN=-0.00563	CM=-0.12039
	CENTER OF PRESSURE IS LOCATED AT 0.7453						FX= 0.094	FZ= 0.450	FY= -0.012	TZ= -0.040	TY= -0.399
1-103G	0.005	0.0546	0.0366	0.9102E 05	8.0	0.0	CD= 0.05201	CL= 0.23062	CS=-0.00070	CN= 0.00042	CM=-0.17143
	CENTER OF PRESSURE IS LOCATED AT 0.7276						FX= 0.137	FZ= 0.650	FY= -0.001	TZ= 0.003	TY= -0.548
1-104G	0.004	0.0539	0.0357	0.8987E 05	10.0	0.0	CD= 0.07668	CL= 0.30556	CS= 0.00372	CN=-0.00365	CM=-0.23118
	CENTER OF PRESSURE IS LOCATED AT 0.7356						FX= 0.197	FZ= 0.842	FY= 0.009	TZ= -0.025	TY= -0.733
1-105G	0.004	0.0535	0.0352	0.8922E 05	12.0	0.0	CD= 0.11003	CL= 0.38780	CS= 0.00394	CN=-0.00183	CM=-0.29600
	CENTER OF PRESSURE IS LOCATED AT 0.7359						FX= 0.279	FZ= 1.056	FY= 0.010	TZ= -0.012	TY= -0.927
1-106G	0.004	0.0532	0.0346	0.8853E 05	14.0	0.0	CD= 0.14807	CL= 0.46563	CS= 0.00287	CN= 0.00031	CM=-0.36503
	CENTER OF PRESSURE IS LOCATED AT 0.7485						FX= 0.370	FZ= 1.253	FY= 0.007	TZ= 0.002	TY= -1.152
1-107G	0.004	0.0528	0.0342	0.8794E 05	16.0	0.0	CD= 0.19856	CL= 0.55759	CS= 0.00544	CN= 0.00295	CM=-0.44178
	CENTER OF PRESSURE IS LOCATED AT 0.7478						FX= 0.491	FZ= 1.486	FY= 0.013	TZ= 0.019	TY= -1.379
1-108G	0.004	0.0523	0.0335	0.8706E 05	18.0	0.0	CD= 0.25546	CL= 0.64827	CS= 0.00371	CN= 0.00506	CM=-0.52670
	CENTER OF PRESSURE IS LOCATED AT 0.7573						FX= 0.620	FZ= 1.701	FY= 0.009	TZ= 0.033	TY= -1.642
1-109G	0.004	0.0516	0.0326	0.8587E 05	20.0	0.0	CD= 0.31941	CL= 0.73716	CS= 0.00127	CN= 0.00759	CM=-0.61593
	CENTER OF PRESSURE IS LOCATED AT 0.7680						FX= 0.757	FZ= 1.890	FY= 0.003	TZ= 0.048	TY= -1.906
1-110G	0.004	0.0503	0.0310	0.8374E 05	22.0	0.0	CD= 0.41377	CL= 0.84145	CS=-0.00125	CN= 0.01232	CM=-0.70281
	CENTER OF PRESSURE IS LOCATED AT 0.7515						FX= 0.939	FZ= 2.067	FY= -0.002	TZ= 0.074	TY= -2.031
1-111G	0.004	0.0491	0.0295	0.8176E 05	24.0	0.0	CD= 0.47353	CL= 0.91876	CS=-0.00492	CN= 0.01517	CM=-0.79270
	CENTER OF PRESSURE IS LOCATED AT 0.7681						FX= 1.027	FZ= 2.162	FY= -0.010	TZ= 0.087	TY= -2.248
1-112G	0.003	0.0483	0.0286	0.8049E 05	26.0	0.0	CD= 0.57544	CL= 1.01216	CS=-0.00753	CN= 0.02164	CM=-0.85291
	CENTER OF PRESSURE IS LOCATED AT 0.7426						FX= 1.218	FZ= 2.327	FY= -0.015	TZ= 0.121	TY= -2.297
1-113G	0.003	0.0478	0.0281	0.7965E 05	28.0	0.0	CD= 0.65173	CL= 1.08172	CS=-0.00622	CN= 0.02811	CM=-0.93763
	CENTER OF PRESSURE IS LOCATED AT 0.7435						FX= 1.359	FZ= 2.452	FY= -0.012	TZ= 0.154	TY= -2.464
1-114G	0.003	0.0484	0.0287	0.8057E 05	26.0	0.0	CD= 0.56126	CL= 0.99824	CS=-0.00976	CN= 0.02210	CM=-0.85983
	CENTER OF PRESSURE IS LOCATED AT 0.7520						FX= 1.190	FZ= 2.299	FY= -0.020	TZ= 0.124	TY= -2.328
1-115G	0.004	0.0492	0.0297	0.8192E 05	24.0	0.0	CD= 0.47838	CL= 0.91653	CS=-0.00794	CN= 0.01670	CM=-0.78523
	CENTER OF PRESSURE IS LOCATED AT 0.7609						FX= 1.043	FZ= 2.167	FY= -0.017	TZ= 0.097	TY= -2.215
1-116G	0.004	0.0516	0.0326	0.8587E 05	20.0	0.0	CD= 0.31817	CL= 0.73416	CS=-0.00347	CN= 0.01095	CM=-0.61166
	CENTER OF PRESSURE IS LOCATED AT 0.7658						FX= 0.755	FZ= 1.884	FY= -0.008	TZ= 0.070	TY= -1.888
1-117G	0.004	0.0527	0.0341	0.8775E 05	16.0	0.0	CD= 0.19483	CL= 0.55222	CS=-0.00034	CN= 0.00626	CM=-0.44391
	CENTER OF PRESSURE IS LOCATED AT 0.7594						FX= 0.480	FZ= 1.458	FY= -0.000	TZ= 0.041	TY= -1.407

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13. ABSTRACT

The equations relating the forces and moments exerted on a body by the magnet fields produced by the MIT-NASA Prototype Magnetic Balance are presented. A computer program which will derive the aerodynamic coefficients for a body using these relations is listed along with a sample output. A preliminary procedure for aligning the axis of the magnetic suspension system with a reference axis is detailed. A procedure for determining dynamic stability derivatives is outlined.

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